

Pivotal Persuasion*

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December 22, 2018

Abstract

A sender seeks to persuade a group of heterogeneous voters to adopt an action. We analyze the sender's information-design problem when the collective decision is made through a majority vote and voting for the action is personally costly. We show that the sender can exploit the heterogeneity in voting costs by privately communicating with the voters. Under the optimal information structure, voters with lower costs are more likely to vote for the sender's preferred action when it is the wrong choice than those with higher costs. The sender's preferred action is therefore adopted with a higher probability when private communication is allowed than when it is not. Nevertheless, the sender's preferred action cannot be adopted with probability one if no voter, as a dictator, is willing to vote for it without being persuaded.

Keywords: Bayesian Persuasion, Information Design, Private Persuasion, Strategic Voting.

JEL Classification Codes: D72, D83

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1 Introduction

This paper considers an information-design problem of a sender who seeks to persuade a group of voters to support an action when voting for this action is costly for the voters. For example, consider a legislative body that consists of members from two political parties: D and R. Since party R does not have enough votes, it needs some support from party D to pass a bill. A party D legislator will pay a political cost if he votes in favor of the policy proposed by party R.¹ To succeed, the leader of party R, therefore, must convince enough legislators from party D that the value of the bill is sufficiently large that it is worthwhile to pay the political cost. Other examples of this type of group persuasion include a candidate urging his or her supporters to turn out to vote, or a CEO trying to convince a board of directors to support a project. In some countries, supporters of opposition candidates may be subject to intimidation at the polls, and a director who supports a CEO's pet project may suffer a loss in reputation.

In our model, a group of N voters must decide between two alternatives: a and b . Action b is the default choice; action a is chosen only if it receives at least $K < N$ votes. There are two states, labeled A and B . Action a is the right choice in state A . Specifically, all voters prefer a in state A but are indifferent between a and b in state B . While action a is a weakly better choice, a voter must incur an extra cost to vote for a . The cost must be paid regardless of the outcome of the vote, and it is heterogeneous across voters. A sender tries to lobby the voters to vote for a . We model the lobbying process as a persuasion game (Kamenica and Gentzkow 2011). First, the sender commits to a signal structure that maps each state to a probability distribution of private signal profiles. The chosen signal structure is publicly observed by all voters. Then, a vector of private signals is drawn according to the signal structure. Finally, voting takes place after each voter observes her own private signal. Following the literature, we focus on the sender's favorite equilibrium.

If the voters knew the state, our model would become a standard threshold public-good game (Palfrey and Rosenthal 1984). Voters would vote against a in state B , and there would be multiple equilibria in state A . In the best equilibrium for the sender, exactly K voters would vote for a , while the rest would free ride. When the voters do not know the state, the sender can increase the

¹On August 10, 1964, two senators, Ernest Gruening and Wayne Morse, voted against a joint congressional resolution allowing the president "to take all necessary steps, including the use of armed force, to assist any member or protocol state of the Southeast Asia Collective Defense Treaty requesting assistance in defense of its freedom." In the 1968 election, they were both defeated, which was believed to be due to their votes. See <http://thehill.com/opinion/campaign/365818-congress-has-forgotten-how-put-principle-above-politics>

probability of a being chosen by designing a signal structure judiciously. On one hand, the sender wants the signals to be noisy, such that the voters would vote for a in state B with some probability. On the other hand, the signals must be sufficiently accurate. Because of the incentives to free ride, the sender must convince a voter not only that voting for a is the right thing to do (i.e., the state is A) but also that her vote is needed.

As a benchmark, we consider the case where the sender is restricted to public communication. In our model, this amounts to requiring voters' signals to be perfectly correlated. In this case, the best the sender can do is to target the voter with the K -th lowest voting cost. If this voter is convinced by a public signal to vote for a , then the $K - 1$ voters with lower voting costs will also be convinced to vote for a . Hence, when restricted to public communication, our model is equivalent to one where the voter with the K -th lowest voting cost is a dictator.² The solution to the latter problem is well known since [Kamenica and Gentzkow \(2011\)](#). The optimal signal should obfuscate the states so that, conditional on voting for a , the voter with the K -th lowest voting cost is indifferent between voting for a and voting for b .

In general, there is no reason why the sender must conduct all communication in public. In fact, interest groups often lobby legislators in private. In our model, the sender can use private communication to his advantage. Under the optimal information structure, every vote for a is pivotal in equilibrium. That is, conditional on receiving a signal that will lead her to vote for a , a voter knows that exactly $K - 1$ other voters also receive signals that will lead them to vote for a . But, because multiple groups of K voters may vote for a in equilibrium, a voter who knows that her vote is pivotal for a will not be able to infer which group of $K - 1$ other voters are voting for a . This allows the sender to fully exploit the heterogeneity in voting costs. Under the optimal information structure, a voter with a lower voting cost will have a strictly higher probability of voting for a in state B than a voter with a higher voting cost. In comparison, when communication is public, the K voters with the lowest voting costs will all vote for a with the same probability in state B . Thus, the sender can get a chosen with a higher probability when private communication is allowed than when it is not. However, the sender's ability to manipulate the outcome is limited. In particular, the sender cannot get a adopted with probability one if no voter, as a dictator, is willing to vote for a without any extra information. Our results sharply contrast with the case where voters' payoffs depend only on the collective decision and not on their individual votes. In such a model, a voter is indifferent about how she votes whenever her vote is not pivotal, and the sender can exploit this

²We call a voter as a dictator if the collective decision coincides with her vote regardless of other voters' choices.

property to get a selected with a probability arbitrarily close to one, so long as the voters strictly prefer a in state A . From the perspective of institutional design, the lesson is that a voting body is less susceptible to manipulation when voters must pay a price for their individual votes.

Throughout the paper, we follow the literature and assume that the sender can commit to any information structure. This assumption enables us to highlight key intuitions. In reality, the set of feasible information structures may be limited. From the perspective of the voters, our exercise can be viewed as a “robust approach” to evaluate the manipulability of majority voting rules.

Related Literature. Our work belongs to the growing literature about information design and Bayes correlated equilibria (BCE) developed by [Bergemann and Morris \(2013, 2016a,b, 2009\)](#) and [Taneva \(2014\)](#). As in the aforementioned papers, we use a linear programming approach to characterize the sender’s optimal implementable information structures. Our contribution is to analyze the benefit of private persuasion in a strategic voting environment.

Our paper is also related to the Bayesian persuasion literature, which uses a belief-based approach to analyze information-design problems. In a one-sender-one-receiver model, [Kamenica and Gentzkow \(2011\)](#) show that the sender’s problem is simply to split the receiver’s belief about the state subject to the standard Bayes’ rule (or Bayes plausibility condition).³ [Mathevet, Perego, and Taneva \(2017\)](#) propose a belief-based and epistemic approach to information design in a multiple-agent setting. [Alonso and Câmara \(2016b\)](#) consider public persuasion in a Bayesian persuasion game between one sender and multiple voters. They show that, when there are more than two payoff-dependent states, the optimal public signal may give rise to multiple distinct winning coalitions that adopt the sender’s preferred action. [Schnakenberg \(2015\)](#) obtains a similar result in a cheap-talk model. In our model, because there are only two payoff-dependent states, and voters have common preference in each state, the optimal public signal always targets the K voters who are the easiest to persuade, and it is strictly dominated by private signals. [Wang \(2015\)](#) considers private persuasion with independent and identically distributed (i.i.d.) signals and shows that public persuasion performs as well as private persuasion. We allow the sender to use any correlated signals and highlight its contribution to the advantage of private persuasion. [Bardhi and Guo \(2018\)](#) investigate a persuasion game under unanimous rule when voters’ preferences are correlated.

The comparison between private and public persuasion has also been studied in non-voting environments. [Inostroza and Pavan \(2018\)](#) study information design in a global game where a pol-

³See also [Brocas and Carrillo \(2007\)](#) and [Rayo and Segal \(2010\)](#).

icy maker designs a stress test to minimize the chance of regime change. They also show that private persuasion dominates public persuasion as in our paper.⁴ In their model, private persuasion dominates as it makes it harder for the agents to coordinate on a successful attack. In [Arieli and Babichenko \(2016\)](#), a sender promotes a product to a group of heterogeneous consumers, each of whom makes his own adoption decision. In contrast to our setting, there is no strategic interaction among consumers. Also see [Farrell and Gibbons \(1989\)](#) and [Goltsman and Pavlov \(2011\)](#) for comparison between public and private communication in multiple-audience cheap-talk models.

There is a large body of literature on information aggregation through strategic voting since the seminal contributions of [Austen-Smith and Banks \(1996\)](#), [Feddersen and Pesendorfer \(1997, 1998\)](#), and [Li, Rosen, and Suen \(2001\)](#).⁵ The key insight of this literature is that, despite voting simultaneously, a rational voter votes as if she knows that her vote is pivotal. In our model, under the optimal information structure, every vote for a is pivotal. Nevertheless, the sender can still increase the influence of voters whose preferences are closer to his own.

A set of papers study the impact of voting cost on turnout and electoral outcomes; see [Palfrey and Rosenthal \(1985\)](#), [Borgers \(2004\)](#), [Feddersen and Sandroni \(2006\)](#), [Taylor and Yildirim \(2010a,b\)](#), and [Krishna and Morgan \(2012\)](#). Unlike these models, ours assumes that the voters' voting cost depends on how they vote instead of whether they vote. [Downs \(1957\)](#) and [Fiorina \(1976\)](#) are the first to study voting models where voters are motivated by non-instrumental considerations. [Morgan and Várdy \(2012\)](#) consider a model where voters have both instrumental and expressive motives. They show that even a weak expressive motive may significantly affect equilibrium voting behavior and the optimal size of a voting body. In our model, voters pay a fixed cost for voting for the sender's preferred action. [Levy \(2007\)](#) analyzes how the reputational cost for voting for an action may depend on whether individual votes are revealed to the public.

Finally, we show that in the presence of voting cost, the sender finds it optimal to induce minimum winning coalitions to get his preferred action adopted. This result is consistent with the prediction of a large number of papers on coalition formation and vote buying. Examples include [Baron and Ferejohn \(1989\)](#), [Koehler \(1975\)](#) and [Shepsle \(1974\)](#).

⁴Also see [Goldstein and Huang \(2016\)](#), who focus on public persuasion.

⁵This literature has been enriched in many dimensions: [Gerardi and Yariv \(2007\)](#) compare various voting rules when voters are allowed to deliberate before casting their votes. [Jackson and Tan \(2013\)](#) allow voters to consult experts before voting, and examine how information revelation varies with different voting rules and with signal precision of the experts. [Li \(2001\)](#), [Persico \(2004\)](#), [Gerardi and Yariv \(2008\)](#) and [Cai \(2009\)](#) assume that voters (or committee members) endogenously collect their information individually.

The rest of the paper is organized as follows. We present the model in section 2; section 3 formalizes the sender’s information-design problem and provides some preliminary results; section 4 characterizes an optimal information structure; section 5 discusses the robustness of our results; section 6 concludes; and omitted proofs are in the appendix.

2 Model

Voting Environment. A group of N voters needs to decide between two alternatives (or actions), denoted by $x \in \{a, b\}$. The collective decision is made according to a non-unanimity K -majority rule whereby alternative a is chosen if and only if it receives K or more votes, and $K < N$. Voters are labeled as $i = 1, \dots, N$. Denote $\mathcal{N} \equiv \{1, \dots, N\}$ as the set of voters.

Payoffs. A voter’s payoff depends on (i) the collective decision x , (ii) her vote $y \in \{a, b\}$, and (iii) the state of the world $\omega \in \{A, B\}$. Specifically, if b is chosen by the group, each voter i receives 0 in either state A or B . If the collective decision is a , she receives 1 when the state is A and 0 when the state is B . In addition, she pays a cost of $c_i > 0$ for voting for a , regardless of the state or the voting outcome.⁶ Her payoff function is

$$u_i(x, y, \omega) = \mathbb{1}_{\{x=a, \omega=A\}} - c_i \mathbb{1}_{\{y=a\}}, \quad (1)$$

where $\mathbb{1}_{\{z\}}$ is an indicator function which equals 1 if condition $\{z\}$ is true and 0 otherwise. We assume that $c_i < 1$, so that each voter i is willing to vote for a if she is a dictator and knows that the state is A . The cost can be interpreted as a reputation loss for supporting an unpopular alternative, or, when the voters are legislators, the political cost for voting against the interests of the constituents. We assume that

$$0 < c_1 < \dots < c_N < 1.$$

Voter 1 is therefore the easiest to persuade to vote for a , while voter N is the hardest.

Voters are uncertain about the state. We assume that they share a common prior belief

$$\mu_0 = \Pr(\omega = A) = 0.5.$$

⁶Here, we normalize the (state-and-outcome-independent) cost of voting for b to be zero. Our results remain as long as the b -voting cost is lower than the cost of voting for a for each voter. We assume that the voting cost is state-independent. In section 5, we discuss how our results will change when the voting cost may depend on the state.

Each voter seeks to maximize her expected payoff. We assume that $c_K > 0.5$, so that fewer than K voters are willing to vote for a as a dictator at the prior belief.⁷

The aforementioned voting environment and voters' payoffs together define a *base voting game*. Specifically, the base voting game consists of (i) a K -majority rule, (ii) a state space $\{A, B\}$, (iii) the set of voters \mathcal{N} , (iv) for each voter $i \in \mathcal{N}$, an action space $\{a, b\}$ and a payoff function $u_i(\cdot)$ specified in expression (1), and (v) the common prior μ_0 .

Information Structure. A sender, who prefers a regardless of the state, seeks to influence the voting outcome by controlling the information of the voters. An *information structure* consists of a set of finite spaces $\{S_i\}_{i \in \mathcal{N}}$ and a pair of probability distributions $\{\pi(\cdot|\omega)\}_{\omega=A,B} \subseteq \Delta(S)$, where S_i denotes the set of possible signals to voter i and $S = \times_{i \in \mathcal{N}} S_i$. We write π for $\{\pi(\cdot|\omega)\}_{\omega=A,B}$. An information structure (S, π) specifies a state-contingent distribution over *signal profiles* $s = (s_1, s_2, \dots, s_N)$, where s_i denotes the signal observed by voter i and $\pi(s|\omega)$ denotes the probability that signal profile s is realized in state ω .

A base voting game and an information structure (S, π) jointly induce a *Bayesian voting game*, which proceeds as follows. First, voters observe the information structure (S, π) . Then, state ω and signal profile s are realized. Finally, the voters simultaneously vote after observing their private signals. A mixed strategy of voter i is a function $\sigma_i : S_i \rightarrow [0, 1]$ that maps each of her private signal s_i to a probability of voting for a . A strategy profile is a Bayesian Nash equilibrium if each σ_i maximizes the expected payoff of voter i given the strategy profile of other voters σ_{-i} .

3 The information-Design Problem

The sender's objective is to design an information structure to maximize the probability of a being the collective decision. In general, the voting game may contain multiple Bayesian Nash equilibria. Throughout the paper, we assume that the sender can select his favorite equilibrium.⁸ Since both the sender and voters prefer a in state A , the selection assumption amounts to letting the sender choose the equilibrium with the highest selection probability of a in state B , which is

⁷The assumption of the prior belief being 0.5 is without loss of generality. Our analysis extends to the settings where voters have heterogeneous prior beliefs. See [Alonso and Câmara \(2016a\)](#) and [Shimoji \(2016\)](#) for persuasion models with such heterogeneity.

⁸The equilibrium selection assumption in our setting, requiring coordination among multiple voters, is stronger than the one in [Kamenica and Gentzkow \(2011\)](#), where only one receiver is involved. See [Bergemann and Morris \(2009\)](#) and [Mathevet, Perego, and Taneva \(2017\)](#) for discussion on equilibrium selection in information-design problems.

also the worst equilibrium to the voters among all equilibria in which a is chosen in state A .

3.1 Public Persuasion

To establish a benchmark, we first consider the case where the sender is restricted to using public signals. Formally, an information structure (S, π) is *public* if (i) $S_i = S_j$ for any $i, j \in \mathcal{N}$ and (ii) $\pi(s|\omega) = 0$ for $\omega = A, B$ and for any s such that $s_i \neq s_j$ for some $i, j \in \mathcal{N}$. For convenience, we drop the subscript i and use S to denote the set of public signals and s to denote a typical element of S . Write $\pi(s|\omega)$ for the probability that s is realized in state ω . Fix a public information structure (S, π) . Let $\tilde{S} \subseteq S$ be the set of signals that result in a being chosen in the sender's favorite equilibrium. For each s , the sender can have K or more votes for a only if

$$\pi(s|A) \geq c_K (\pi(s|A) + \pi(s|B)).$$

In this case, voters' public posterior belief about the state being A is at least c_K , and voters $1, \dots, K$ find it optimal to vote for a . Summing over $s \in \tilde{S}$, we must have

$$\sum_{s \in \tilde{S}} \pi(s|A) \geq c_K \sum_{s \in \tilde{S}} (\pi(s|A) + \pi(s|B)).$$

Since $\sum_{s \in \tilde{S}} \pi(s|A) \leq 1$, $\sum_{s \in \tilde{S}} (\pi(s|A) + \pi(s|B)) \leq 1/c_K$. Thus, the selection probability of a must be less than $1/c_K$ under public persuasion. It is easy to see that the bound can be achieved by a binary public signal $S = \{a, b\}$ with $\pi(a|A) = 1$ and $\pi(a|B) = (1 - c_K)/c_K$, and an equilibrium in which voters 1 to K vote for a if and only if they observe signal a while voters $K + 1$ to N always vote for b . Intuitively, the sender's optimal strategy is to target voters 1 to K , who are the easiest to persuade. Since voter K is the hardest to persuade among this group, a would be chosen if and only if voter K is willing to vote for a as a dictator.

3.2 Simplifying the Sender's Problem

We now return to the general case. We first show that, by the revelation principle, we can focus on *obedience* strategies and binary information structures. An information structure (S, π) is *binary* if $S_i = \{a, b\}$ for each $i \in \mathcal{N}$. We can view $s_i \in \{a, b\}$ as a recommendation to vote for s_i . A strategy σ_i is *obedient* if it follows the recommendation (i.e., $\sigma_i(a) = 1 - \sigma_i(b) = 1$). We say that a voter votes obediently if she follows an obedience strategy, and an equilibrium is obedient if every voter's equilibrium strategy is obedient.

Lemma 1. For any information structure (S, π) , if σ is a Bayesian Nash equilibrium in the voting game induced by (S, π) and $\tau(\sigma)$ is the state-dependent distribution over vote profiles induced by σ , then every voter voting obediently is a Bayesian Nash equilibrium in the voting game induced by the binary information structure $(\{a, b\}^N, \tau(\sigma))$.

Lemma 1 is a consequence of the revelation principle (Myerson 1986).⁹ It says that if a distribution of vote profiles is an equilibrium outcome under some information structure, then it must also be the outcome of an *obedience* equilibrium under a binary information structure where each vote profile is recommended with the profile's realization probability in the original equilibrium. Thus, we can characterize the voting behavior that yields the highest selection probability of a under any information structure by considering only obedience equilibria. Henceforth, unless otherwise stated, all information structures are binary and, when it does not cause confusion, we drop the signal space and denote a binary information structure simply by π .

A signal profile is called *a-winning* if K or more voters are recommended to vote for a . It is a *minimum a-winning profile* if exactly K voters are recommended to vote for a . We denote

$$S_a \equiv \{s \in \{a, b\}^N \mid |\{i \in \mathcal{N} : s_i = a\}| \geq K\}$$

as the set of *a-winning signal profiles*,¹⁰ and

$$S_a^* \equiv \{s \in S_a \mid |\{i \in \mathcal{N} : s_i = a\}| = K\}$$

as the set of *minimum a-winning signal profiles*. Similarly, the sets of *b-winning* and *minimum b-winning signal profiles*, are denoted by S_b and S_b^* , respectively. For $x \in \{a, b\}$ and $i \in \mathcal{N}$, the sets

$$S_{i,x} \equiv \{s \in S_x \mid s_i = x\} \text{ and } S_{i,x}^* \equiv \{s \in S_x^* \mid s_i = x\}$$

denote the set of x -winning signal profiles and the set of minimum x -winning signal profiles where voter i is recommended to vote for x . We say that voter i is *pivotal* in signal profile s if $K - 1$ voters other than voter i are recommended to vote for a in s . For each (minimum) x -winning signal profile, we call the set of voters who are recommended to vote for x as the corresponding (*minimum*) x -winning coalition.

⁹Also see Taneva (2014) and Bergemann and Morris (2016a, 2009) for recent application of this idea to the persuasion problem.

¹⁰We use $|X|$ to denote the total number of elements in a finite set X .

Fix an information structure π . Consider a voter i who is recommended to vote for a with a strictly positive probability. Conditional on $s_i = a$ and assuming all voters other than i vote obediently, voter i 's expected payoff for voting for a is

$$-c_i + \frac{\sum_{s \in S_{i,a}} \pi(s|A)}{\sum_{s \in S: s_i = a} (\pi(s|A) + \pi(s|B))} \quad (2)$$

where the first term is the cost of voting for a , and the second term is, conditional on i receiving recommendation a , the probability of the state being A and the collective decision being a . If she votes for b , her payoff is

$$\frac{\sum_{s \in S_{i,a} \setminus S_{i,a}^*} \pi(s|A)}{\sum_{s \in S: s_i = a} (\pi(s|A) + \pi(s|B))}. \quad (3)$$

which represents, conditional on i receiving recommendation a and actually voting for b , the probability that the state being A and the collective decision being a . It is therefore optimal for voter i to vote obediently for a if

$$-c_i \sum_{\omega=A,B} \sum_{s \in S: s_i = a} \pi(s|\omega) + \sum_{s \in S_{i,a}^*} \pi(s|A) \geq 0. \quad (4)$$

Similarly, it is optimal for voter i to vote obediently for b if

$$-c_i \sum_{\omega=A,B} \sum_{s \in S: s_i = b} \pi(s|\omega) + \sum_{s \in S_{i,b}^*} \pi(s|A) \leq 0. \quad (5)$$

Voting obediently is, therefore, a Bayesian Nash equilibrium if π satisfies constraints (4) and (5) for every $i \in \mathcal{N}$.

By Lemma 1, the sender's problem is to choose a probability distribution over signal profiles to maximize the selection probability of a subject to the voters' obedience constraints. For any π , let

$$Q(\pi) \equiv 0.5 \sum_{s \in S_a} (\pi(s|A) + \pi(s|B))$$

denote the total probability of implementing a when voters vote obediently. Let Π denote the set of binary information structures. The sender's information-design problem (Problem P-0) is

$$\max_{\pi \in \Pi} Q(\pi) \quad (\text{P-0})$$

subject to constraints (4) and (5) for every $i \in \mathcal{N}$. The following propositions establish some basic properties of the optimal information structure.

Proposition 1. *If π is a solution to Problem (P-0), then $\sum_{s \in S_a} \pi(s|A) = 1$.*

Proposition 1, which says that a is always selected in state A under any optimal solution π , results from the fact that both the sender and voters prefer a in state A .

Let

$$\Pi^{piv} \equiv \{\pi \in \Pi \mid \pi(s|A) = \pi(s|B) = 0, \forall s \notin S_a^* \cup \{(b, \dots, b)\}\}$$

denote the set of information structures that assign zero probability to non-minimum a -winning signal profiles and to non-unanimous b -winning signal profile.

Proposition 2. *There exists a solution π to Problem (P-0) with $\pi \in \Pi^{piv}$.*

Proposition 2 says that there is an optimal solution to the sender's problem where a is chosen always by exactly K votes and b is chosen always unanimously. Under this optimal information structure, when a voter is recommended to vote for a , she knows that exactly $K - 1$ other voters are also recommended to vote for a ; in other words, she knows that her vote is pivotal.¹¹ Intuitively, because a voter has to pay a cost for voting for a regardless of the collective decision, she will vote for a only if she is convinced that there is a sufficiently high probability that the state is A and she is pivotal. It is therefore optimal for the sender to select a with a minimum number of votes. Note that when an optimal solution $\pi \in \Pi^{piv}$, the obedience constraint to vote for b , condition (5), is not binding for every i , since a voter who is recommended to vote for b is never pivotal.

4 The Optimal Information Structure

By Proposition 2, we can, without loss of generality, restrict the sender's choice set to Π^{piv} . Assuming that voter i is recommended to vote for a with strictly positive probability, her obedience constraint for a , condition (4) becomes

$$\frac{\sum_{s \in S_{i,a}^*} \pi(s|A)}{\sum_{s \in S_{i,a}^*} \pi(s|A) + \sum_{s \in S_{i,a}^*} \pi(s|B)} \geq c_i \quad (6)$$

where the left-hand side is voter i 's *pivotal belief*, i.e., the posterior belief conditional on being pivotal. Thus, voter i is willing to vote for a only if her pivotal belief exceeds c_i .

For the analysis that follows, it is convenient to write $l_i \equiv c_i / (1 - c_i)$ as the likelihood ratio of the cutoff pivotal belief. Since l_i is strictly increasing in c_i , our assumption that c_i increases in i with

¹¹ Furthermore, when a is not always selected ($Q(\pi) < 1$), a will never receive more than K votes under any optimal information structure. See Corollary 1.

$c_K > 0.5$ means that

$$0 < l_1 < \dots < l_N,$$

and $l_K > 1$. Since the obedience constraint for voting for b is not binding, we can write the sender's problem as Problem (P-1)

$$\max_{\pi \in \Pi^{piv}} \sum_{s \in S_a^*} \pi(s|B) \quad (\text{P-1})$$

subject to

$$\sum_{s \in S_a^*} \pi(s|A) = 1; \quad (7)$$

$$\sum_{s \in S_{i,a}^*} \pi(s|A) \geq l_i \sum_{s \in S_{i,a}^*} \pi(s|B) \quad \forall i \in \mathcal{N}; \quad (8)$$

where the “budget” constraint (7) is a consequence of Propositions 1 and 2, and the obedience constraint (8) is immediate given condition (6). Hereafter, we let Q_B denote the value of Problem (P-1).

4.1 A Three-Voter Case

We illustrate the main features of an optimal information structure with a simple example where $N = 3$ and $K = 2$. In this case, there are three minimum a -winning signal profiles: aab , aba , and baa .¹² The sender's Problem (P-1) is

$$\max_{\pi \in \Pi^{piv}} \{\pi(aab|B) + \pi(aba|B) + \pi(baa|B)\} \quad (9)$$

subject to

$$l_1[\pi(aab|B) + \pi(aba|B)] \leq \pi(aab|A) + \pi(aba|A), \quad (10)$$

$$l_2[\pi(aab|B) + \pi(baa|B)] \leq \pi(aab|A) + \pi(baa|A), \quad (11)$$

$$l_3[\pi(aba|B) + \pi(baa|B)] \leq \pi(aba|A) + \pi(baa|A), \quad (12)$$

and

$$\pi(aab|A) + \pi(aba|A) + \pi(baa|A) = 1. \quad (13)$$

¹²We use aab to denote the signal profile that recommends voters 1 and 2 to vote for a and voter 3 to voter for b .

Recall that when the sender is restricted to using public signals, the best he can do is to design a signal that induces voters 1 and 2 to vote for a with probability one in state A and with probability $1/l_2$ in state B . In our current framework, this amounts to the binary information structure satisfying

$$\begin{aligned}\pi(baa|A) &= 0 & ; & & \pi(baa|B) &= 0; \\ \pi(aba|A) &= 0 & ; & & \pi(aba|B) &= 0; \\ \pi(aab|A) &= 1 & ; & & \pi(aab|B) &= \frac{1}{l_2}.\end{aligned}$$

Note that the obedience constraint of voter 1 is slack as $l_1 < l_2$.

The sender can increase the probability of a being selected in state B by adjusting the information structure in the following way:

1. Decrease $\pi(aab|A)$ by ϵ and increase $\pi(baa|A)$ by ϵ . Constraint (11) is still binding and constraints (10) and (12) are slack if $\epsilon > 0$ is sufficiently small.
2. Increase $\pi(aba|B)$ by $\epsilon/l_3 > 0$. Now constraints (11) and (12) are both binding. As long as ϵ is sufficiently small, constraint (10) will still be slack.

The information structure now satisfies

$$\begin{aligned}\pi(baa|A) &= \epsilon & ; & & \pi(baa|B) &= 0; \\ \pi(aba|A) &= 0 & ; & & \pi(aba|B) &= \frac{\epsilon}{l_3}; \\ \pi(aab|A) &= 1 - \epsilon & ; & & \pi(aab|B) &= \frac{1}{l_2}.\end{aligned}$$

The selection probability of a in state B increases by ϵ/l_3 . The shift allows the sender to take advantage of the slackness of voter 1's obedience constraint. Now, in state B voter 1 belongs to two a -winning coalitions that occur with a strictly positive probability. Note that it is crucial that voter 3 observes only the recommendation for her but not those for the other voters. If voter 3 knew that the recommendations are aba (and not baa), she would not vote for a .¹³

The following proposition describes the optimal information structure in the three-voter case.

Proposition 3. *Under the optimal information structure, the probability that a is selected in state B is*

$$Q_B = \min\left(\frac{1}{l_2} \frac{l_2 + l_3}{l_1 + l_3}, 1\right).$$

¹³In fact, aba is a perfect signal of state B in the sense that $\pi(aba|A) = 0$ and $\pi(aba|B) > 0$.

When $Q_B < 1$, any optimal information structure must satisfy

$$\begin{aligned}\pi(baa|A) &= \frac{l_3}{l_2} \frac{l_2 - l_1}{l_1 + l_3} \quad ; \quad \pi(baa|B) = 0; \\ \pi(aba|A) &= 0 \quad ; \quad \pi(aba|B) = \frac{1}{l_2} \frac{l_2 - l_1}{l_1 + l_3}; \\ \pi(aab|A) &= \frac{l_1}{l_2} \frac{l_3 + l_2}{l_1 + l_3} \quad ; \quad \pi(aab|B) = \frac{1}{l_2}.\end{aligned}$$

The information structure in Proposition 3 can be obtained by choosing ϵ in the aforementioned procedure such that constraint (10) binds. Note that voter 3, who is the hardest to persuade, votes for a with positive probability. Furthermore, it is straightforward to check that

$$Q_B = \pi(aab|B) + \pi(aba|B) > \pi(aab|B) + \pi(baa|B) > \pi(baa|B) + \pi(aba|B).$$

That is, an easier-to-persuade voter is more likely to vote for a in state B , and voter 1 is pivotal whenever a is selected in state B .

As is common in the literature, the sender finds it optimal to obfuscate the states. A more remarkable feature of the optimal information structure is that the sender also *obfuscates the winning coalitions*. By using private signals, the sender can conceal from a voter who else is being recommended to vote for a .¹⁴ The obfuscation allows the sender to fully take advantage of the voters' heterogeneous preferences—all three obedience constraints for voting for a are binding under the optimal information structure.

The gain from public to private persuasion is

$$\pi(aba|B) = \frac{l_2 - l_1}{l_1 l_2 + l_2 l_3}, \tag{14}$$

which decreases in l_3 and l_1 and vanishes as either $l_1 \rightarrow l_2$ or $l_3 \rightarrow \infty$. The slackness in voter 1's obedience constraint under the optimal public information structure disappears at the first limit; voter 3 becomes almost impossible to persuade at the second limit, and the slackness in voter 1's obedience constraint has a very small effect.

When it is less than 1, the probability that a is adopted in state B is

$$Q_B = \pi(aba|B) + \pi(aab|B) = \frac{l_2 + l_3}{l_2 (l_1 + l_3)},$$

¹⁴The randomization over multiple winning coalitions also implies that some winning coalitions contain voters who are not among the easiest to persuade. The feature is also present in an example in Alonso and Câmara (2016b). Despite the similarity of the results, the mechanisms behind them are very different.

which is decreasing in l_i for every i . This is intuitive. Increasing l_i is equivalent to increasing a voter's threshold of doubt, c_i . To make the recommendation convincing, the signal must be more accurate about the realized state, leaving the sender less room to obfuscate states for voter i . Since all voters vote for a with strictly positive probability in the optimal information structure, Q_B is decreasing in every l_i . Since $l_3 > l_2 > 1$, $Q_B < 1$ when $l_1 > 1$ or when l_3 goes to infinity. Thus, the selection probability of a can equal to one only if voter 1 is willing to vote pivotally for a ex ante and voter 3 is not too difficult to persuade.

4.2 General Case

When there are N voters, there are $\binom{N}{K}$ minimum a -winning signal profiles. Instead of dealing with many minimum a -winning signal profiles, it is more convenient to reformulate the sender's problem in terms of the probabilities of each voter receiving a recommendation to vote for a .

Let $\alpha = (\alpha_1, \dots, \alpha_N)$ and $\beta = (\beta_1, \dots, \beta_N)$ be N -vectors. Suppose that there is a feasible solution π to Problem (P-1) such that, for all $i \in \mathcal{N}$,

$$\alpha_i = \sum_{s \in \mathcal{S}_{i,a}^*} \pi(s|A); \quad (15)$$

$$\beta_i = \sum_{s \in \mathcal{S}_{i,a}^*} \pi(s|B). \quad (16)$$

Substituting conditions (15) and (16) into voter i 's obedience constraint (8) yields

$$l_i \beta_i \leq \alpha_i. \quad (17)$$

Since each minimum a -winning coalition includes K voters voting for a , the probabilities that a is chosen in states A and B are, respectively,

$$\frac{1}{K} \sum_{i=1}^N \alpha_i = 1, \quad (18)$$

$$\frac{1}{K} \sum_{i=1}^N \beta_i \leq 1. \quad (19)$$

The equality in condition (18) follows conditions (7) and (15). Since the probability that voter i belongs to a minimum a -winning coalition must be lower than the probability that a is selected, we have

$$\alpha_i \in [0, 1] \quad \forall i; \quad (20)$$

$$\beta_i \in \left[0, \frac{1}{K} \sum_{j=1}^N \beta_j \right] \quad \forall i. \quad (21)$$

Since, under an optimal information structure, every vote for a is pivotal, we shall refer to a voter's probability of being recommended to vote for a as her *pivotal probability*. The discussion above means that if (α, β) are the voters' pivotal probabilities in state A and B , respectively, of a feasible solution to the sender's problem (P-1), then (α, β) must satisfy condition (17) for all $i \in \mathcal{N}$, as well as conditions (18), (19), (20), and (21). The following lemma establishes the converse.

Lemma 2. *For any (α, β) that satisfy conditions (17), (18), (19), (20), and (21), there exists a feasible solution π to Problem (P-1) such that conditions (15) and (16) hold for all $i \in \mathcal{N}$.*

Lemma 2 means that we can restate Problem (P-1) directly in terms of pivotal probabilities. Formally, consider Problem (P-2):

$$Q_B = \max_{\alpha, \beta} \frac{1}{K} \sum_{i=1}^N \beta_i \quad (\text{P-2})$$

subject to conditions (17), (18), (19), (20), and (21). We will refer to Problem (P-2) as the reduced form of Problem (P-1).

The following lemma is immediate.

Lemma 3. *If π is a solution to Problem (P-1), then (α, β) defined by conditions (15) and (16) is a solution to Problem (P-2). Conversely, if (α, β) is a solution to Problem (P-2), then there exists a solution π to Problem (P-1) such that conditions (15) and (16) hold.*

Intuitively, voters use the pivotal probabilities in state A as “input” to “produce” pivotal probabilities in state B . For each unit of α_i , voter i can produce $1/l_i$ unit of β_i up to an “output ceiling” of $\sum_{j=1}^N \beta_j / K$. The sender's task is to divide K units of pivotal probabilities in state A among voters to maximize total production of pivotal probabilities in state B .

It is inefficient to allocate all K units of input to voters 1 to K (as would be the case if the sender uses a public signal designed to persuade voter K). In this case the maximum output consistent with each β_i not exceeding the output ceiling is to have each voter 1 to K produce $1/l_K$ unit of β_i , meaning that some of the input allocated to voters 1 to $K - 1$ is wasted.

To characterize the solution to Problem (P-2), we first solve a relaxed problem and then relate the solution to Problem (P-2) to the solution of this relaxed problem. Consider Problem (P-3):

$$U(Y) = \frac{1}{K} \max_{\alpha, \beta} \sum_{i=1}^N \beta_i \quad (\text{P-3})$$

$$\text{s.t. } l_i \beta_i \leq \alpha_i \forall i \quad (22)$$

$$\sum_i \alpha_i = K, \alpha_i \in [0, 1]; \beta_i \in [0, Y] \forall i. \quad (23)$$

Problem (P-3) discards constraint (19) in Problem (P-2), and replaces the endogenous ceiling $\sum_i \beta_i / K$ in (21) by an exogenous parameter Y in (23). Notice that in an optimal solution to Problem (P-2), Q_B appears in constraint (21), which makes Q_B a fixed point of $U(\cdot)$.

Since l_i increases in i , the optimal solution to Problem (P-3) is straightforward. First, allocate as many units of input as possible to voter 1, who is the most efficient in producing β_1 , until $\alpha_1 = \min(1, l_1 Y)$.¹⁵ Then, allocate as many units of input as possible to voter 2 until $\alpha_2 = \min(1, l_2 Y)$, and so on. Continue the process until either all K units of input are allocated, or each voter i has received either one or $l_i Y$ units of input. Let

$$i^*(Y) \equiv \max \left\{ i \in \mathcal{N} \left| \sum_{j=1}^i \min(1, l_j Y) < K \right. \right\}$$

denote the last voter i with $\alpha_i = \min(1, l_i Y)$. The value of Problem (P-3) is

$$U(Y) = \frac{1}{K} \left\{ \sum_{j=1}^{i^*(Y)} \min\left(Y, \frac{1}{l_j}\right) + \mathbb{1}_{\{i^*(Y) < N\}} \left(K - \sum_{j=1}^{i^*(Y)} \min(1, l_j Y) \right) \frac{1}{l_{i^*(Y)+1}} \right\}, \quad (24)$$

where $\mathbb{1}_{\{i^*(Y) < N\}}$ equals one if $i^*(Y) < N$ and zero otherwise.

When $i^*(Y) < N$, all inputs are exhausted at the end of the allocation process. In this case, following the discussion above, the unique optimal solution to Problem (P-3) is to set, for each voter i ,

$$\hat{\alpha}_i(Y) = \begin{cases} \min(1, l_i Y) & \text{if } i \leq i^*(Y), \\ K - \sum_{j=1}^{i^*(Y)} \min(1, l_j Y) & \text{if } i = i^*(Y) + 1, \\ 0 & \text{if } i > i^*(Y) + 1, \end{cases}$$

and

$$\hat{\beta}_i(Y) = \begin{cases} \min\left(Y, \frac{1}{l_i}\right) & \text{if } i \leq i^*(Y), \\ \frac{K - \sum_{j=1}^{i^*(Y)} \min(1, l_j Y)}{l_i} & \text{if } i = i^*(Y) + 1, \\ 0 & \text{if } i > i^*(Y) + 1. \end{cases}$$

Proposition 4. *In problem (P-2), $Q_B < 1$ if and only if $U(1) < 1$. Furthermore, $Q_B < 1$ if either $l_1 > 1$ or l_{K+1} is sufficiently large.*

The first part of Proposition 4 provides a necessary and sufficient condition for the value of Problem (P-2) to be strictly less than one. To establish the sufficiency, notice that any feasible solution to Problem (P-2) is also feasible in Problem (P-3) when $Y = 1$. Hence, if $U(1) < 1$, the value

¹⁵Since β_i must not exceed Y , raising α_i beyond $l_i Y$ cannot further increase β_i .

of Problem (P-2) must also be strictly less than 1. We prove the necessity in the Appendix. In the three-voter example, $Q_B < 1$ when $l_1 > 1$ or l_3 is sufficiently large. The second part of Proposition 4 shows that the same is true in general. Intuitively, the sender is trying to choose an information structure that maximizes the impact of voter 1, who is the easiest to persuade in the group. If voter 1 is unwilling to vote for a as a dictator with probability one, then there is no way the sender can make the group select a with probability one. Furthermore, as we shall show in Proposition 5, under the optimal information structure, at least one voter who is harder to persuade than voter K must also be persuaded to vote for a with positive probability in state B . Even when l_1 is very small, Q_B will still be less than one when this voter is very hard to persuade.

The following proposition describes some key properties of the optimal solution to Problem (P-2).

Proposition 5. *When $Q_B < 1$ and $i^*(Q_B) < N$, then $(\alpha^*, \beta^*) \equiv (\hat{\alpha}(Q_B), \hat{\beta}(Q_B))$ is the unique solution to (P-2) and has the following properties:*

1. $l_i \beta_i^* = \alpha_i^* \forall i$,
2. $\alpha_i^* = \min(1, l_i Q_B)$ if $\alpha_j^* > 0$ for some $j > i$,
3. β_i^* decreases in i , $\beta_1^* = Q_B$, and $\beta_{K+1}^* > 0$.

When $Q_B < 1$ and $i^*(Q_B) < N$, the unique solution to Problem (P-3) with $Y = Q_B$ is also a unique solution to Problem (P-2). The sender persuades voters $i = 1, \dots, i^*(Q_B) + 1$. Three properties of the optimal solution are intuitive. First, each voter's obedience constraint for voting for a is binding. If the constraint for some voter i were not binding, the sender could either increase β_i or reduce α_i (and increase some other α_j). Second, the sender should allocate as many units of input as possible to voters with the lowest l_i 's. If some input is allocated to voter j (i.e., $\alpha_j^* > 0$), then either the input limit (20) or output ceiling (21) must be reached for any voter $i < j$. Third, voters who are easier to persuade have higher pivotal probabilities in state B than voters who are harder to persuade. In particular, the pivotal probability of voter 1 in state B is equal to Q_B . Hence, as in the three-voter example, voter 1 must belong to every minimum a -winning coalition in state B that is assigned with positive probability. Lastly, voter $K + 1$ has a strictly positive pivotal probability in state B . Since each minimum a -winning profile has only K votes for a , an optimal information structure must assign strictly positive probabilities to multiple minimum a -winning profiles.¹⁶

¹⁶When $Q_B = 1$ or $i^*(Q_B) = N$, there may be multiple optimal solutions. When the voters are sufficiently easy to

Corollary 1. *If π is a solution to Problem (P-0) with $Q(\pi) < 1$, then $\pi \in \Pi^{piv}$.*

Proposition 2 shows that there is an optimal information structure $\pi \in \Pi^{piv}$. Proposition 5 implies that when $Q_B < 1$, every optimal information structure must belong to Π^{piv} . To see this, consider some information structure π that is a feasible solution to the sender's problem and which assigns a strictly positive probability to a signal profile s' that is either a non-minimum a -winning profile or a non-unanimous b -winning profile.¹⁷ If it is the former, we can transfer the probability assigned to s' to a minimum a -winning profile s'' where the set of voters recommended to vote for a is a subset of the voters recommended to vote for a in s' . If it is the latter, we can transfer the probability assigned to s' to the profile where every voter is recommended to vote for b . In either case, this will result in a new signal structure π' that is feasible, with the same selection probability of a in state B as π , but where the obedience constraint for voting for a is slack for some voter. (See the proof of Proposition 2 for details.) By Proposition 5, π' cannot be optimal. Since π and π' lead to the same selection probability of a in state B , π cannot be optimal as well.

The following proposition describes how Q_B is affected by the preferences of voters and the majority requirement.

Proposition 6. *In Problem (P-2), when $Q_B < 1$ and $i^*(Q_B) < N$,*

1. Q_B is strictly decreasing in l_i if $\alpha_i^* > 0$,
2. $Q_B \rightarrow 1/l_K$ as either $l_1 \rightarrow l_K$ or $l_{K+1} \rightarrow \infty$, and
3. Q_B decreases in K .

Part 1 of Proposition 6 says that Q_B increases when any voter who votes for a with a strictly positive probability becomes easier to persuade. Recall that the sender can achieve $Q_B = 1/l_K$ with public signals. Part 2 says that the gain from persuading with private signals disappears either if voters 1 to K have the same preferences, or if voters $K + 1$ to N are very hard to persuade. In the first case, as in the three-voter example, when $l_1 \rightarrow l_K$, the obedience constraint (17) becomes binding for every voter i under the optimal public information structure. Thus, there is no room

persuade, the sender may achieve $Q_B = 1$ without targeting the voters who are the easiest to persuade. When some voters are very easy to persuade and some very hard to persuade (e.g., some l_i close to zero and some l_i very large), $Q_B < 1$ and $i^*(Q_B) = N$. In this case, in every optimal solution $\beta_i = \min(Q_B, 1/l_i)$ for all $i \in \mathcal{N}$, but there may be multiple ways to choose α_i that are optimal.

¹⁷A b -winning profile is non-unanimous if some voters are recommended to vote for a .

for manipulation. In the second case, recall that to raise Q_B above $1/l_K$, at least one voter $i > K$ must be persuaded to vote for a . When l_{K+1} goes to infinity, voters $K + 1, \dots, N$ become impossible to persuade. Lastly, part 3 says that Q_B decreases in the number of votes needed to select a . It is well known that a group may choose a large majority requirement to encourage information acquisition when information is costly (see Li, 2001). Our results suggest that doing so may also have the benefit of limiting the influence of interested third parties. The result holds even when $l_{K+1} = l_K$. The reason is twofold. First, a large K forces the sender to persuade harder-to-convince voters, making it more difficult to build a -winning coalitions. Second, a larger K forces the sender to persuade more voters, reducing his ability to target different coalitions of voters, which is consistent with the observation in Alonso and Câmara (2016b).

Propositions 5 and 6 reveal the extent to which majority voting is susceptible to manipulation when voters have voting costs. The result contrasts sharply with the case where voters' payoffs depend solely on the outcome and not on their votes. Consider a canonical strategic voting model with purely instrumental motives as in Austen-Smith and Banks (1996). There are two states, A and B , and two actions, a and b . When a is the collective choice, each voter i receives $x_i > 0$ in state A and 0 in state B . When b is the collective choice, each voter i receives 0 in state A and $y_i > 0$ in state B . It is easy to see that the following is a strict perfect Bayesian Nash equilibrium:

When the state is B , with probability $1 - \epsilon$ all voters are asked to vote for a , and with probability ϵ the sender randomly selects one of the minimum b -winning profiles with equal probability. When the state is A , the sender randomly selects one of the minimum a -winning profiles with equal probability.

In this equilibrium, a is chosen with probability $1 - 0.5\epsilon$. Intuitively, the sender can get a selected with probability arbitrarily close to one by exploiting a non-pivotal voter's indifference between voting for or against a .¹⁸ In our model, since the voting cost must be paid regardless of the outcome of the vote, a voter is unwilling to vote for a unless she believes that there is a sufficiently high chance that her vote is pivotal and the state is A .

The comparison above suggests an advantage of making the voting process more transparent. When a voting body is subject to outside manipulation, the reputational concern of voters prevents the collective decision from being fully manipulated. A secret ballot, which makes it hard to infer how each individual voter votes, weakens the reputational concern.

¹⁸Also see Bardhi and Guo (2018).

5 Discussion

We have made a number of assumptions to simplify the analysis. In this subsection, we discuss what happens if some of these assumptions are relaxed.

Instrumental Preference. We have assumed that a and b yield the same payoff in state B for all voters. Suppose when the collective decision is b , each voter receives 0 in state A and $d > 0$ in state B . Voter i 's obedience constraints for voting for a and b , respectively, will then become

$$-c_i \sum_{\omega=A,B} \sum_{s \in S: s_i=a} \pi(s|\omega) - d \sum_{s \in S_{i,a}^*} \pi(s|B) + \sum_{s \in S_{i,a}^*} \pi(s|A) \geq 0 \quad (25)$$

and

$$-c_i \sum_{\omega=A,B} \sum_{s \in S: s_i=b} \pi(s|\omega) - d \sum_{s \in S_{i,b}^*} \pi(s|B) + \sum_{s \in S_{i,b}^*} \pi(s|A) \leq 0. \quad (26)$$

The sender's information design problem is to

$$\max_{\pi \in \Pi} Q(\pi) \quad (\text{P-0}')$$

subject to constraints (25) and (26) for every $i \in \mathcal{N}$.

Compared with constraints (4) and (5), constraints (25) and (26) contain an extra term that captures the payoff d that voter i must forego when voting for a results in alternative a being chosen in state B . A voter therefore may be reluctant to vote for a if she believes that she may be pivotal in state B . This makes constraint (25) harder to hold but constraint (26) easier to hold. In state A , the sender will continue to induce exactly K voters to vote for a , as this is the only way the sender can motivate a voter to vote for a . Hence, Proposition 1 continues to hold. In state B , while the sender may try to induce $K + 1$ voters to vote for a to avoid making any voter pivotal, there is no reason to induce more $K + 1$ votes for a . We thus have the following proposition. Let S_a^{**} denote the set of a -winning signal profiles in which exactly $K + 1$ voters vote for a , and let $S_{i,a}^{**}$ denote the subset of S_a^{**} with $s_i = a$.

Proposition 7. *There exists a solution π to Problem (P-0') with $\pi(s|A) = 0$ for any $s \notin S_a^* \cup \{(b, \dots, b)\}$ and $\pi(s|B) = 0$ for any $s \notin S_a^* \cup S_a^{**} \cup \{(b, \dots, b)\}$.*

The proof of Proposition 7 is omitted as it is almost identical to that of Proposition 2. Let Π^* denote the set of information structures that put zero probability to $s \in S_a^* \cup \{(b, \dots, b)\}$ in state A and $s \in S_a^* \cup S_a^{**} \cup \{(b, \dots, b)\}$ in state B . Proposition 7 means that we can restrict the choice set of the sender to Π^* .

For each $\pi \in \Pi^*$ and every $i \in \mathcal{N}$, let

$$\gamma_i = \sum_{s \in S_{i,a}^{**}} \pi(s|B), \quad (27)$$

which is the total probability assigned to signal profiles in which $K + 1$ voters, including voter i , are recommended to vote for a when the state is B . Write ψ_i for $(c_i + d) / (1 - c_i)$. The reduced form of the sender's problem (Problem P-2') is

$$Q_B = \max_{\alpha, \beta, \gamma} \frac{1}{K} \sum_{i=1}^N \beta_i + \frac{1}{K+1} \sum_{i=1}^N \gamma_i \quad (\text{P-2}') \quad (28)$$

subject to constraints (18), (19), (20), (21), and

$$\psi_i \beta_i + l_i \gamma_i \leq \alpha_i \quad \forall i, \quad (28)$$

$$\frac{1}{K+1} \sum_{i=1}^N \gamma_i \leq 1; \quad (29)$$

$$\gamma_i \in \left[0, \frac{1}{K+1} \sum_{j=1}^N \gamma_j \right] \quad \forall i. \quad (30)$$

Constraint (28) is obtained by substituting conditions (15), (16) and (27) into voter i 's obedience constraint. Constraint (29) is needed because each a -winning coalition induced by $s \in S_a^{**}$ includes $K + 1$ voters voting for a . Constraint (30) holds because the probability that voter i belongs to a non-minimum a -winning coalition induced by a signal profile $s \in S_a^{**}$ must be lower than the probability that a is selected due to the support of a non-minimum a -winning coalition. The same argument in Lemmas 2 can be used to show that for any $\gamma = (\gamma_1, \dots, \gamma_n)$ that satisfies constraints (29) and (30), there exists a probability distribution over $S_{i,a}^{**}$ that generates γ . Hence, we can directly work with the reduced form as in the baseline model.

Problem (P-2') differs from Problem (P-2) in that, in addition to deciding how many units of "input" α_i should be allocated to voter i , the sender must also decide whether the input should be used to "produce" β_i or γ_i . On the one hand, γ_i is less costly to produce than β_i as voter i 's vote is not pivotal in a non-minimum a -winning coalition. On the other hand, each unit of β_i is worth more than γ_i in the objective function. Furthermore, raising β_i and γ_i will relax the constraints (19) and (30), respectively, and the shadow values associated of these constraints may be different. Problem (P-2'), therefore, cannot be readily solved.¹⁹ Nevertheless, we can show that some basic properties of the optimal solution in Problem (P-2) will carry over to Problem (P-2').

¹⁹Solving Problem (P-2') is beyond the scope of this paper.

Proposition 8. *If $Q_B < 1$ in Problem (P-2'), then any solution $(\alpha^*, \beta^*, \gamma^*)$ must satisfy the following properties:*

1. *if $\alpha_i^* < 1$ and either $\beta_i^* < \frac{1}{K} \sum_{i=1}^N \beta_i^*$ or $\gamma_i^* < \frac{1}{K+1} \sum_{i=1}^N \gamma_i^*$ for some i , then for all i , $\psi_i \beta_i^* + l_i \gamma_i^* = \alpha_i^*$;*
2. *if $\alpha_j^* > 0$ for some j , then either $\alpha_i^* = 1$, or $\beta_i^* = \frac{1}{K} \sum_{i=1}^N \beta_i^*$, or $\gamma_i^* = \frac{1}{K+1} \sum_{i=1}^N \gamma_i^*$ for all $i < j$.*

The two properties in Proposition 8 are weaker versions of the first two properties in Proposition 5. Proposition 8 says that if there is a voter i with α_i^* less than one, and either β_i^* or γ_i^* less than the upper bound, then the obedience constraint of every voter i for voting for a must be binding. The second says that if voter j is voting for a with strictly positive probability, then for every voter i who is easier to persuade than voter j , it must be that either α_i^* , β_i^* , or γ_i^* is at the upper bound.

Voting Cost. We assume that a voter must pay a cost for voting for a regardless of the state. But in some cases, the voting cost could be state dependent. For example, a director voting for a controversial project may suffer a bigger reputation loss if the project fails. If voter i pays c_i for voting for a only when the state is B , then voter i 's obedience constraint would become

$$-c_i \sum_{s \in S: s_i = a} \pi(s|B) + \sum_{s \in S_{i,a}^*} \pi(s|A) \geq 0. \quad (31)$$

Both Propositions 1 and 2 will continue to hold, as it remains in the interest of the sender to select a with the minimum of votes in state B and to convince voters that their votes are pivotal in state A .

Lastly, we want to emphasize that Proposition 2 depends crucially on the assumption that there is no state-contingent cost for voting for b when the state is A . For the sake of illustration, consider an extreme case where voters care only about voting “correctly” and not at all about the collective decision. Specifically, imagine that each voter i pays $c_i \in (0, 1)$ when she votes for a in state B and $d_i \in (0, 1)$ when she votes for b in state A . Given the recommendation signal, a voter simply wants to minimize the expected cost of voting incorrectly. Thus, voter i obedience constraint for voting for a becomes

$$c_i \sum_{s \in S: s_i = a} \pi(s|B) \leq d_i \sum_{s \in S: s_i = a} \pi(s|A).$$

Therefore, it is optimal for the sender to recommend voter i to vote for a with probability one in state A and with probability $\min\{d_i/c_i, 1\}$ in state B . The sender's task is to coordinate the votes in state B to maximize the selection probability of a .

6 Conclusion

This paper considers the information-design problem of a sender who tries to persuade a group of voters. We characterize the optimal information structure and explain why private persuasion strictly outperforms public persuasion.

We assume that the sender has no private information about the state before choosing the information structure. It is natural to ask what happens if the assumption is relaxed, especially in our voting context where an interested party or politician engaged in persuading has superior information about the state to voters. [Alonso and Câmara \(2016a\)](#) use the results from [Alonso and Câmara \(2018\)](#) to argue that, if the sender knows the state before persuasion, then the sender's favorite equilibrium in this informed-sender game is an equilibrium in which all informed senders pool on the same information structure used by an uninformed sender. Their result extends to the current paper. That is, if the information designer knows the true state before choosing the information structure, there is an equilibrium in this informed-designer game where all types of senders pool on the same signal as the uninformed sender. It is also natural to ask what happens if voters have private information, as in [Kolotilin, Mylovanov, Zapechelnyuk, and Li \(2017\)](#), [Kolotilin \(2018\)](#), [Bergemann, Bonatti, and Smolin \(2018\)](#) and [Guo and Shmaya \(2017\)](#). We leave this extension for future work.

A Appendix: Omitted Proofs

Proof of Lemma 1. The proof follows directly from the Revelation Principle as in Myerson (1986), Bergemann and Morris (2016a) and Taneva (2014) and is omitted. \square

Proof of Proposition 1. Suppose that $\pi(s|A) > 0$ for some $s \in S_b$. Because $c_i \in (0, 1), \forall i$, the sender can always set $\pi(s|A) = 0, \forall s \in S_b$ and increase the probabilities that $\sum_{s \in S_a} \pi(s|A)$ without violating voters' obedience constraints. \square

Proof of Proposition 2. For any $s \in S_a \setminus S_a^*$, we can pick some $s' \in S_a^*$ such that for all $i \in \mathcal{N}$, $s'_i = a$ only if $s_i = a$. Hence, we can construct a function $f(s) : S_a \setminus S_a^* \rightarrow S_a^*$ such that $f_i(s)$, the i -th component of $f(s)$, is equal to a only if $s_i = a$. In words, $f(s)$ is a minimum a -winning profile where every voter who is recommended to vote for a is also recommended to vote for a in s .

Let f^{-1} denote the inverse mapping of f . Suppose π is an optimal solution to Problem (P-0). By Proposition 1, $\sum_{s \in S_a} \pi(s|A) = 1$. Define π' as follows:

$$\pi'(s|A) \equiv \begin{cases} \pi(s|A) + \sum_{s' \in f^{-1}(s)} \pi(s'|A) & \text{if } s \in S_a^*; \\ 0 & \text{if } s \notin S_a^*; \end{cases}$$

$$\pi'(s|B) \equiv \begin{cases} \pi(s|B) + \sum_{s' \in f^{-1}(s)} \pi(s'|B) & \text{if } s \in S_a^*; \\ \sum_{s' \in S_b} \pi(s'|B) & \text{if } s = (b, \dots, b); \\ 0 & \text{if } s \notin S_a^* \cup \{(b, \dots, b)\}. \end{cases}$$

Thus, π' is formed from π by shifting, in both states A and B , the probability weight from each $s \in S_a \setminus S_a^*$ to $f(s)$ and, in state B , from each $s \in S_b$ to (b, \dots, b) . Furthermore, it is easy to check that

$$\pi'(s|A) = 0 \quad \forall s \notin S_a^* \cup \{(b, \dots, b)\}. \quad (32)$$

Because under π' , in state A , a is always recommended to exactly K voters, a voter who is recommended to vote for b knows that her vote is never pivotal when the state is A . This means that, for each voter i , $\sum_{s \in S_{i,b}^*} \pi'(s|A) = 0$ and the obedience constraint for voting for b , condition (5), is satisfied under π' .

We now turn to the obedience constraint for voting for a , condition (4), which can be rewritten as:

$$-c_i \sum_{s \in S: s_i = a} \pi(s|B) - c_i \sum_{s \in S_{i,a} \setminus S_{i,a}^*} \pi(s|A) + (1 - c_i) \sum_{s \in S_{i,a}^*} \pi(s|A) \geq 0. \quad (33)$$

By supposition, π satisfies (4) for all voters i . By construction,

$$\sum_{s \in S_{i,a} \setminus S_{i,a}^*} \pi'(s|A) \leq \sum_{s \in S_{i,a} \setminus S_{i,a}^*} \pi(s|A),$$

and

$$\sum_{s \in S_{i,a}^*} \pi'(s|A) \geq \sum_{s \in S_{i,a}^*} \pi(s|A).$$

Furthermore, since $f_i(s) = a$ only if $s_i = a$, for each voter i ,

$$\sum_{s \in S: s_i = a} \pi'(s|B) \leq \sum_{s \in S: s_i = a} \pi(s|B).$$

It follows that π' also satisfies (4) for all voters i . Thus, π' is feasible for Problem (P-0). By construction, the probability that a is selected under π' is the same as under π . Hence, π' is also optimal. This proves the Proposition. \square

Proof of Proposition 3. Since the purpose of the 3-voter example is to illustrate the idea of the more general N -voter model, we use the notations introduced in section 4.2 to prove this proposition. Note that in the three-voter case, $i^*(Q_B) < 3$ for any Q_B as $l_1 > 0$. Hence, when $Q_B < 1$, by Proposition 5, $\beta_1^* = Q_B$ and $\alpha_2^* = \min(1, l_2 Q_B) = 1$. It follows that $\pi(baa|B) = \pi(aba|A) = 0$. The solution is the only one that makes (10), (11), (12), and (13) binding. It follows that $Q_B = \min\{\pi(aba|B) + \pi(aab|B), 1\}$. \square

Proof of Lemma 2. We show that for any α such that conditions (18) and (20) hold, there exists a $\pi(s|A)$ such that condition (15) holds.

Let $h = \binom{N}{K}$ be the number of minimum a -winning signal profiles. It is convenient to state the proposition in matrix form. Let $s^{[1]}, \dots, s^{[h]}$ be an order of the minimum a -winning signal profiles. Let W be a $N \times h$ matrix with

$$W_{ij} = \begin{cases} 1 & \text{if } s_i^{[j]} = a, \\ 0 & \text{if } s_i^{[j]} = b. \end{cases}$$

We need to show that there is a vector

$$\pi_A = \begin{pmatrix} \pi_{1A} \\ \vdots \\ \pi_{hA} \end{pmatrix},$$

such that $\sum_{i=1}^h \pi_{iA} = 1$ and $\pi_{iA} \in [0, 1]$, and

$$W\pi_A = \alpha \equiv \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix}.$$

Suppose by way of contradiction that no such π_A exists. By Farkas' lemma, there exists a vector

$$\lambda = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{pmatrix},$$

such that

$$W^T \lambda \geq 0; \tag{34}$$

$$\alpha^T \lambda < 0. \tag{35}$$

Then each row of W^T corresponds to a signal profile where exactly K voters observe a , including one that begins with $K + 1$ ones followed by $N - K$ zeros. Thus, condition (34) implies that

$$\sum_{i=1}^K \lambda_i \geq 0.$$

Since the player ordering can be arbitrarily rearranged, we can assume without loss of generality that λ_i is ascending in i . Hence

$$\begin{aligned} & \min_{x_i} \sum_{i=1}^N \lambda_i x_i \text{ s.t. } x_i \in [0, 1] \forall i, \sum_{i=1}^N x_i = K, \\ & = \sum_{i=1}^K \lambda_i \geq 0. \end{aligned}$$

Since $\alpha_i \in [0, 1]$ for all i , and $\sum_{i=1}^N \alpha_i = K$,

$$\sum_{i=1}^N \lambda_i \alpha_i \geq 0,$$

which contradicts condition (35).

Similarly, one can show that for any $\beta = (\beta_1, \dots, \beta_n)$ subject to conditions (19) and (21), there exists $\pi(s|B)$ such that condition (16) holds. \square

Proof of Proposition 4. We first show that if $Q_B < 1$ then $U(1) < 1$. To do so, suppose that $U(1) \geq 1$, and we shall show that $Q_B = 1$. There are two cases. In the case where $U(1) = 1$, the corresponding optimal solution to Problem (P-3) with $Y = 1$ is feasible to Problem (P-2) with a value $Q_B = 1$.

Consider the case where $U(1) > 1$, and denote (α^*, β^*) as an optimal solution. Note that both the objective function and constraints in Problem (P-3) are linear in (α, β) (for any Y). That is, if both (α, β) and (α', β') are feasible with values ν and ν' , respectively, then for any $\lambda \in [0, 1]$, $(\lambda\alpha + (1 - \lambda)\alpha', \lambda\beta + (1 - \lambda)\beta')$ is feasible with value $\lambda\nu + (1 - \lambda)\nu'$. We know that (α, β) where

$$\alpha_i = \begin{cases} 1 & \text{if } i \leq K \\ 0 & \text{if } i > K \end{cases}, \beta_i = \begin{cases} 1/l_K & \text{if } i \leq K \\ 0 & \text{if } i > K, \end{cases}$$

is a feasible choice to Problem (P-3) with $Y = 1$ with value equal to $1/l_K < 1$. Therefore, there exists a $\lambda \in [0, 1]$ such that $(\lambda\alpha + (1 - \lambda)\alpha^*, \lambda\beta + (1 - \lambda)\beta^*)$ is a feasible solution to Problem (P-3) with $Y = 1$ with value exactly equal to 1. This solution is also feasible for Problem (P-2), meaning that $Q_B = 1$. \square

To show that $Q_B < 1$ if $l_1 > 1$, note that

$$l_1 \sum_{i \in \mathcal{N}} \beta_i \leq \sum_{i \in \mathcal{N}} l_i \beta_i \leq \sum_{i \in \mathcal{N}} \alpha_i = K. \quad (36)$$

The second inequality in condition (36) is obtained by summing condition (17) over i . Dividing condition (36) by K and substituting Q_B for $\sum_{i \in \mathcal{N}} \beta_i / K$, we have

$$l_1 Q_B \leq 1.$$

We now show that $Q_B < 1$ if l_{K+1} is sufficiently large. Since $l_K > 1$, and $\min(1/l_i, 1) \leq 1$,

$$U(1) \leq \frac{1}{K} \left\{ K - 1 + \frac{1}{l_K} + \frac{K}{l_{K+1}} \right\}.$$

Hence,

$$\begin{aligned} \lim_{l_{K+1} \rightarrow \infty} U(1) &\leq \lim_{l_{K+1} \rightarrow \infty} \frac{1}{K} \left\{ K - 1 + \frac{1}{l_K} + \frac{K}{l_{K+1}} \right\} \\ &= \frac{1}{K} \left\{ K - 1 + \frac{1}{l_K} \right\} \\ &< 1. \end{aligned}$$

Proof of Proposition 5. Suppose that $Q_B < 1$, $i^*(Q_B) < N$ and (α, β) is a solution to Problem (P-2). Notice that (α, β) must be feasible in Problem (P-3) when $Y = Q_B$, leading to $U(Q_B) \geq Q_B$. We want

to show that $U(Q_B) = Q_B$. Suppose, by way of contradiction, that $U(Q_B) > Q_B$ in Problem (P-3). This is possible only if $U(Q_B) > 1$; otherwise, Q_B cannot be the value of Problem (P-2). Since $U(1) \geq U(Q_B)$, if $U(Q_B) > 1$, then $U(1) > 1$, which, by Proposition 4, implies that $Q_B = 1$, a contradiction. Hence, in this case an optimal solution to Problem (P-2) must also be optimal in Problem (P-3) with $Y = Q_B$. Since we have already shown in text that when $i^*(Q_B) < N$, that $(\hat{\alpha}(Q_B), \hat{\beta}(Q_B))$ is the unique solution to Problem (P-3) with $Y = Q_B$, it must also be the unique solution to Problem (P-2).

The first and second properties of the solution follow immediately from the definition of $(\hat{\alpha}(Q_B), \hat{\beta}(Q_B))$. The third property that β_i^* decreases in i follows from the first two properties and the fact that l_i is increasing. By the first two properties, we have

$$\beta_1^* = \frac{\alpha_1^*}{l_1} = \min\left(\frac{1}{l_1}, Q_B\right).$$

We want to show that $\beta_1^* = Q_B < 1/l_1$. Suppose not. Then $\beta_1^* = \min(\frac{1}{l_1}, Q_B) = \frac{1}{l_1} \leq Q_B$. Since l_i is increasing in i , (α_i^*, β_i^*) would be equal to $(1, 1/l_i)$ for $i \leq K$ and $(0, 0)$ for $i > K$. But then

$$Q_B = \frac{1}{K} \sum_{i=1}^K \frac{1}{l_i} < \frac{1}{l_1},$$

contradicting the supposition that $1/l_1 \leq Q_B$. This proves that $\beta_1^* = Q_B < 1/l_1$. Finally, since $\alpha_1^* = l_1 Q_B < 1$, $\sum_{i=1}^K \alpha_i^* < K$. It then follows from the fact that $\sum_{i=1}^N \alpha_i^* = K$ that α_{K+1}^* and β_{K+1}^* are strictly positive. \square

Proof of Proposition 6. Part 1. Suppose l_i decreases for some i where $\alpha_i^* > 0$. Then voter i 's obedience constraint for voting for a becomes slack. Since by assumption $i^*(Q_B) < N$, we can reduce α_i^* by ϵ , increase $\alpha_{i^*(Q_B)+1}^*$ by ϵ and $\beta_{i^*(Q_B)+1}^*$ by $\epsilon/l_{i^*(Q_B)+1}$.

Part 2. Since $1/l_K$ can always be achieved under public persuasion, $Q_B \geq 1/l_K$. For any $Y > 1/l_K$, $(K - \sum_{j=1}^K \min(1, l_j Y)) < 0$ when l_1 (and therefore also l_2, \dots, l_{K-1}) is sufficiently close to l_K . Hence,

$$\lim_{l_1 \rightarrow l_K} U(Y; l_1, \dots, l_N) < \frac{1}{K} \sum_{i=1}^K \min\left(Y, \frac{1}{l_i}\right) < Y.$$

As we argued in Proposition 5, $U(Q_B) = Q_B$. Since for all $Y > 1/l_K$,

$$\lim_{l_1 \rightarrow l_K} U(Y; l_1, \dots, l_N) < Y,$$

it follows that

$$\lim_{l_1 \rightarrow l_K} Q_B \leq 1/l_K.$$

A similar argument applies when l_{K+1} (and therefore l_{K+2}, \dots, l_N) becomes sufficiently large, i.e.,

$$\lim_{l_{K+1} \rightarrow \infty} Q_B \leq 1/l_K,$$

as for all $Y > 1/l_K$,

$$\begin{aligned} & \lim_{l_{K+1} \rightarrow \infty} U(Y; l_1, \dots, l_N), \\ & \leq \frac{1}{K} \left[\sum_{i=1}^{K-1} \min\left(Y, \frac{1}{l_j}\right) + \frac{1}{l_K} + \lim_{l_1 \rightarrow l_K} \left(K - \sum_{j=1}^K \min(1, l_j) \right) \frac{1}{l_{K+1}} \right], \\ & = \frac{1}{K} \left(\sum_{i=1}^{K-1} \min\left(Y, \frac{1}{l_j}\right) + \frac{1}{l_K} \right) < Y. \end{aligned}$$

Part 3. Suppose that $Q_B < 1$ and $i^*(Q_B) < N$. Recall that $i^*(Q_B) > K$ is the last voter who votes for a with probability one in state A . Suppose K increases by 1. Keeping Q_B constant for now. From Proposition 5, we know that we will allocate the new unit of pivotal probability to voter $i^*(Q_B) + 1$ and, probably, voter $i^*(Q_B) + 2$ (if $i^*(Q_B) + 1 < N$). This will increase the sum of all β_i^* by less than Q_B since, from Proposition 5, β_i is decreasing in i and $\beta_{i^*(Q_B)}^* = 1/l_{i^*(Q_B)} < Q_B$. This means that the new average (now with denominator $K + 1$) will be strictly less than Q_B . Hence Q_B is no longer the solution. Since $U(Y)$ is monotone in Y , the new solution must be smaller. \square

Proof of Proposition 8. Part 1. Suppose, by way of contradiction, that $\alpha_i^* < 1$ and either $\beta_i^* < \frac{1}{K} \sum_{i=1}^N \beta_i^*$ or $\gamma_i^* < \frac{1}{K+1} \sum_{i=1}^N \gamma_i^*$ for some i . If the obedience constraint for a is not binding for some voter j , then the sender can reduce α_j^* by some small $\epsilon > 0$ without violating the constraint. Then, the sender can increase Q_B by raising α_i^* by ϵ and raising either β_i^* or γ_i^* .

Part 2. Note that $l_i < l_j$ and $\psi_i < \psi_j$ for any $i < j$. Suppose for some $j > i$, $\alpha_j^* > 0$, and $\alpha_i^* < 1$, $\beta_i^* < \frac{1}{K} \sum_{i=1}^N \beta_i^*$, and $\gamma_i^* < \frac{1}{K+1} \sum_{i=1}^N \gamma_i^*$. By part 1, $\psi_j \beta_j^* + l_j \gamma_j^* = \alpha_j^*$. Hence, either $\beta_j^* > 0$ or $\gamma_j^* > 0$. Suppose it is the former. For some small $\epsilon > 0$, we can, without violating the obedience constraints for voting for a for voters j and i , reduce β_j^* by ϵ and α_j^* by $\psi_j \epsilon$, and raise α_i^* by $\psi_j \epsilon$ and β_i^* by $\psi_j \epsilon / \psi_i$. Since $\psi_i < \psi_j$, the change will raise Q_B . The case when $\gamma_j^* > 0$ is similar. \square

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