Rewarding Improvements: Optimal Dynamic Contracts with Subjective Evaluation*

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Abstract

We study a T-period contracting problem where performance evaluations are subjective and private. We find that the principal should punish the agent for performing poorly in the future even when the evaluations were good in the past; at the same time, the agent should be given opportunities to make up for poor evaluations in the past with better performance in the future. Optimal incentives are thus asymmetric. Conditional on the same number of good evaluations, an agent whose performance improves over time should be better rewarded than one whose performance deteriorates.

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1 Introduction

Incentive contracts that explicitly tie compensation to objective performance measures are rare. According to MacLeod and Parent (1999), only about one to five percent of U.S. workers receive performance pay in the form of commissions or piece rates. Far more common, especially in positions that require team work, are long-term relational contracts that reward or punish workers on the basis of subjective performance measures that are not verifiable in court. Early work in the literature of subjective evaluation (Bull (1987), MacLeod and Malcomson (1989)) has shown that efficient contracts can be self-enforcing so long as the contracting parties are sufficiently patient and always agree on the performance measure.

Efficiency loss, however, becomes inevitable when the contracting parties may disagree on performance (MacLeod (2003), Levin (2003)). Consider a worker who can either work or shirk. To motivate the worker, the employer promises to pay him a performance bonus if his performance is satisfactory. But as performance is subjective, the employer may claim that the performance is poor to avoid paying the bonus. To avoid this incentive problem, the employer may choose to pay a high wage and use the threat of dismissal to motivate the worker. Because the employer do not gain from the dismissal of a worker, he would have no incentive to cheat.[1] This

[1] An alternative way to keep the employer honest is to let the worker punish the employer when he feels that the employer has cheated (See MacLeod (2003)).

type of efficiency-wage contracts—as they are commonly known in the literature (Levin (2003), Fuchs (2007))—is not fully efficient. As performance is a noisy signal of effort, a hardworking worker may be terminated when the performance turns out to be poor.

The expected efficiency loss, however, can be mitigated in a long-term relationship. For example, in a two-period employment relationship, instead of carrying out the first-period punishment immediately, the employer can "reuse" the punishment by offering to cancel it if the worker performs well in the second period. This idea, first introduced by Abreu, Milgrom, and Pearce (1991) in the context of repeated games, is applied to the contracting setting by Fuchs (2007), who shows that in a T-period contracting game, instead of evaluating the performance of the worker period by period, the employer should punish the worker only if his performance is poor in all T periods.

A crucial assumption in Fuchs (2007) is that the worker's performance is observed only by the employer and not by the worker himself. This is obviously a restrictive assumption. In most situations a worker would have at least some ideas about his contribution. For example, although an analyst may not know exactly how his manager judges the quality of his reports, it is unlikely that his own opinion is completely uncorrelated with that of the manager.[2] The main contribution of this ar-

[2]The same can be said of academic economic research. Though none of us can predict for sure whether an article will be accepted at a particular journal, most of us can tell whether the article has a reasonable chance at journals of similar quality.

ticle is to introduce the agent's self-evaluation into the model and derive the optimal efficiency-wage contract that induces maximum effort when the worker's self evaluation is positively correlated with the employer's evaluation of his performance.

We find that punishing a worker only when the performance is poor in every period is inefficient when the correlation is significant, as a worker who feels that he has been performing well would have little incentives to work in the subsequent periods. To prevent the worker from becoming complacent, the employer needs to punish the worker if he stops performing well (according to the employer's evaluation) after a certain period. But at the same time the agent should be given opportunities to make up for poor performance in the past with better performance in the future.[3] Optimal incentives are thus asymmetric. In particular, we find that the optimal level of punishment depends only on the last period the worker performs well. For example, a worker who performs well only in the last period would receive the same compensation as one who performs well in every period. Conditional on the same number of good evaluations, a worker whose performance improves over time is better rewarded than one whose performance deteriorates.

An important finding of Fuchs (2007) is that the minimum efficiency loss for inducing maximum effort in all periods is independent of the number of periods and, as a result, the per-period efficiency loss would go to zero as the length of the contract goes to infinity. We find that this result continues to hold when the correlation

[3] This is akin to the common practice of letting students make up for their poor midterm grades with better final ones.

between the evaluations of the worker and employer is below a certain threshold. But beyond that threshold, the efficiency loss of a T-period contract increases with the correlation and converges to the loss of T static contracts as the correlation goes to one. For any fixed correlation above the threshold, the per-period efficiency loss decreases in T but is bounded away from zero as T goes to infinity. Some inefficiency, therefore, would remain even when both the employer and worker are patient and a long review horizon is adopted.

The optimal contract in this article rewards a good performance in a latter period better than one in an earlier period. Previous studies have obtained similar findings for different reasons. Lewis and Sappington (1997) show that in the presence of both adverse selection and moral hazard, a good performance in the second period is always rewarded, but a good performance in the first period by an agent who claims to have low ability may be punished. Gershkov and Perry (2009) find that it is optimal in a two-period tournament model to assign a lower weight to first-period outcome when the first-period effort also affects the second-period outcome.

2 Model

We consider a T-period contracting game between a Principal and an Agent. In period 0 the Principal offers the Agent a contract ω. If the Agent rejects the offer, the game ends with each player receiving zero payoff. If the Agent accepts the contract, he is employed for T periods. In each period $t \in \{1, ..., T\}$ of his employment the

Agent decides whether to work ($e_t = 1$) or shirk ($e_t = 0$). The Agent's effort is private and not observed by the Principal. Output is stochastic with the expected output equal to e_t. The effort cost to the Agent is $c(e_t)$, with $c(1) = c > 0$ and $c(0) = 0$. We assume that $c < 1$, so the surplus is maximized when the Agent works in every period.

There is no objective output measure commonly observed by the Principal and the Agent. Instead, each player observes a private binary performance signal at the end of each period. Let $y_t \in \{H, L\}$ and $s_t \in \{G, B\}$ denote the period-t signals of the Principal and Agent, respectively. Neither y_t nor s_t is verifiable by court. Let $\pi(.|e_t)$ denote the joint distribution of (y_t, s_t) conditional on e_t and $\pi(.|e_t, s_t)$ denote the distribution of y_t conditional on e_t and s_t.[4] Both the Principal and the Agent know π. We assume π satisfies the following assumptions:

Assumption 1. $\pi(H|1) = p > \pi(H|0) = q$.

Assumption 2. $\pi(H|1, G) > \max\{\pi(H|1, B), \pi(H|0, G), \pi(H|0, B)\}$.

We say that the Principal considers the Agent's performance in period t as high when $y_t = H$ and low when $y_t = L$, and that the Agent considers his own performance as good when $s_t = G$ and bad when $s_t = B$. Assumption 1 requires that the Principal's evaluation be positively correlated with the Agent's effort. Assumption 2 requires that the Agent's belief about H be the highest when he has worked and observed G. As long as the signals are not independent, we can relabel them so that

[4] Both y_t and s_t are uncorrelated over time.

$\pi(H|1,G) > \pi(H|1,B)$ and $\pi(H|0,G) > \pi(H|0,B)$. Hence, Assumption 2 will be satisfied if $\pi(H|1,G) > \pi(H|0,G)$. Intuitively, the assumption requires that the Agent's evaluation be not "too informative" on the Principal's when $e_t = 0$. It will hold, for example, if the Agent's evaluation is equal to the Principal's evaluation plus noise. We define

$$\rho \equiv \frac{\pi(L|1) - \pi(L|1,G)}{\pi(L|1)} = 1 - \frac{\pi(L|1,G)}{(1-p)}$$

as the correlation coefficient of evaluations conditional on the Agent working; ρ equals 0 when the evaluations are independent and 1 when the evaluations are perfectly correlated.

Both the Principal and the Agent are risk neutral. Were the Principal's signals contractible, the maximum total surplus could be achieved by a standard contract that pays the Agent a high wage when $y_t = H$ and a low wage when $y_t = L$. The problem here is that y_t is privately observed and non-verifiable. If the Principal were to pay the Agent less when he reports L, he would report L regardless of the true signal. To ensure the Principal reporting truthfully, any amount that the Principal deducts from the Agent's compensation when $y_t = L$ must be either destroyed or diverted to a use that does not benefit the Principal.

We call contracts that involve the Principal burning money "efficiency-wage" contracts because they resemble efficiency-wage contracts that pay workers above-market wage until they are fired.[5] Formally, an efficiency-wage contract $\omega\left(B, W, Z^T\right)$

[5]See Fuchs (2007) for such a model.

contains a legally enforceable component (B, W) and an informal punishment agreement Z^T. The enforceable component stipulates that the Principal make an up-front payment B (that can be negative) before period 1 and a final payment $W \geq 0$ after period T.[6] The Agent will receive B in full, but the Principal reserves the right to deduct any amount $Z^T \leq W$ from the final payment and burn it. The exact value of Z^T is governed by an informal punishment strategy $Z^T : \{H, L\}^T \rightarrow \Re_+$ that maps the Principal's information into a positive amount less than W.

In each period t, the Agent decides whether to work. The Agent's history at date t for $t > 1$ consists of his effort choices and the sequence of signals observed in the previous $t-1$ periods, $h^t \equiv e^{t-1} \times s^{t-1}$, where $e^{t-1} \equiv (e_1, \ldots, e_{t-1})$ and $s^{t-1} \equiv (s_1, \ldots, s_{t-1})$. We use h^1 or (e^0, s^0) to denote the null history in period one. Let $\mathcal{H}^t$ denote the set of all period t histories. A strategy for the Agent is a vector $\sigma \equiv (\sigma_1, ..., \sigma_T)$ where $\sigma_t : \mathcal{H}^t \rightarrow \{0, 1\}$ is a function that determines the Agent's effort in period t.

Let $e^T \equiv (e_1, ..., e_T)$ be a sequence of effort choices and $y^T \equiv (y_1, ..., y_T)$ be a sequence of the Principal's signals. Both the Principal and the Agent discount future payoffs by a discount factor $\delta < 1$. A strategy σ induces a probability distribution over e^T and y^T. Let

$$v(Z^T, \sigma) \equiv E\left(Z^T(y^T) + \sum_{t=1}^{T} \delta^{t-1} c(e_t) \middle| \sigma \right)$$

[6]Throughout, all payments regardless of when they actually occur are in terms of the present value evaluated at $t = 1$.

denote the Agent's expected effort and punishment cost under σ. An Agent's strategy σ^* is a best response against Z^T if for all strategies $\sigma \neq \sigma^*$, the expected cost under σ is higher than that under σ^*; that is, if

$$v(Z^T, \sigma^*) \leq v(Z^T, \sigma).$$

The Agent accepts a contract $\omega\left(B, W, Z^T\right)$ if and only if there exists a best response σ^* against Z^T such that $B + W - v(Z^T, \sigma) \geq 0$. A contract $\omega\left(B, W, Z^T\right)$ is optimal for the Principal if there exists an Agent's strategy σ such that $\left(B, W, Z^T, \sigma\right)$ is a solution to the maximization problem:

$$\begin{aligned} \max_{B,W,Z,\sigma} & E\left(-B - W + \sum_{t=1}^{T} \delta^{t-1} e_t \middle| \sigma\right), \\ s.t. \ & \sigma \in \arg\min v(Z^T, \sigma), \\ & Z^T \leq W, \\ & v(Z^T, \sigma) \leq B + W. \end{aligned}$$

Note that neither B nor W affects the Agent's effort decisions. Thus, for any Z^T, the Principal can always set $W \geq Z^T$ and $B = v(Z^T, \sigma) - W$. Hence, we can rewrite the Principal's problem as

$$\begin{aligned} \max_{Z,\sigma} & E\left(\sum_{t=1}^{T} \delta^{t-1}\left(e_t - c\left(e_t\right)\right) - Z^T(y^T) \middle| \sigma\right), \\ s.t. \ & \sigma \in \arg\min v(Z^T, \sigma). \end{aligned}$$

In the following we often refer to Z^T as a contract without specifying B and W.

The Agent works in every period according to σ if for all $t \in \{1, ..., T\}$ and all $h^t \in H^t$, $\sigma_t(h^t) = 1$. We say that Z^T induces maximum effort if working in every period (after any history) is a best response against Z^T. Let $C(Z^T)$ denote the expected money-burning cost of any Z^T that induces maximum effort. A contract is efficient in inducing maximum effort if it has the lowest money-burning cost among all contracts that induce maximum effort. We shall mostly focus on efficient maximum-effort contracts. Such contracts are optimal when effort cost c is sufficiently small.[7]

3 Two-period Contract

A drawback of efficiency-wage contracts is that a positive amount will be destroyed with positive probability even when the Agent works in every period. To see this point, consider the one-period contract.

Proposition 1. *When $T = 1$, any contract that motivates the Agent to work must have an expected money-buring cost of $(1-p)c/(p-q)$ or greater.*

Proof. Working is a best response for the Agent (assuming that the contract has been accepted) if the sum of the effort and money-burning cost is lower when he works; that is, if

$$-\left[pZ^1(H) + (1-p)Z^1(L)\right] - c \geq -\left[qZ^1(H) + (1-q)Z^1(L)\right]. \tag{1}$$

[7] We discuss this issue in Section 3.

Minimizing the expected money-burning loss $C(Z^1) \equiv pZ^1(H) + (1-p)Z^1(L)$, subject to the incentive constraint (1) yields the solution

$$\overline{Z}^1(H) = 0 \text{ and } \overline{Z}^1(L) = \frac{c}{p-q}, \tag{2}$$

with $C(\overline{Z}^1) = (1-p)c/(p-q)$. □

MacLeod (2003) and Levin (2003) are the first to prove this result. Fuchs (2007) demonstrates that when the Agent receives no information about the Principal's signal, the Principal can save money-burning cost by linking the money-burning decisions across periods. Below we use a two-period example to show that some of the stronger results of Fuchs (2007) may not hold when the Agent observes a private signal that is correlated with the Principal's.

Optimal Maximum-Effort Contract

In this section we derive the optimal maximum-effort contract when $T = 2$. Note that any Z^2 that induces maximum effort must satisfy the following two incentive-compatibility constraints:

$$p[Z^2(LH) - Z^2(HH)] + (1-p)[Z^2(LL) - Z^2(HL)] \geq \frac{c}{p-q}; \tag{3}$$

$$\pi(H|1,G)[Z^2(HL) - Z^2(HH)] + \pi(L|1,G)[Z^2(LL) - Z^2(LH)] \geq \frac{\delta c}{p-q}. \tag{4}$$

The first constraint requires that the Agent be better off working in both periods than working only in the second. The second requires that the Agent be better off working in the second period after he has worked and observed G in the first. As a first step to derive the optimal maximum-effort contract, we solve the optimization problem

$$\min_{Z^2} p^2 Z^2(HH) + p(1-p)[Z^2(LH) + Z^2(HL)] + (1-p)^2 Z^2(LL) \tag{5}$$

subject to (3) and (4). We then show that the solution to this problem induces maximum effort. Because any contract that induces maximum effort must satisfy (3) and (4), the solution, if it induces maximum effort, must have the lowest money-burning cost among maximum-effort contracts.

There are two cases to consider. When $\rho < 1 - \delta$, any

$$\widehat{Z}^2(y^2) = \begin{cases} \frac{c}{(p-q)(1-p)} - \frac{p}{1-p}\eta & \text{if } y^2 = LL, \\ \eta & \text{if } y^2 = LH, \\ 0 & \text{if } y^2 = HL, \\ 0 & \text{if } y^2 = HH, \end{cases}$$

with

$$0 \le \eta \le \overline{\eta} \equiv \frac{(1-\rho-\delta)\,c}{(1-\rho)\,(p-q)},$$

is a solution to the minimization problem.

The constraint (4) is not binding at the optimal solution (except when $\eta = \overline{\eta}$). The optimal value of $Z^2(HH)$ is zero as it enters both constraints negatively. An Agent who has performed well in both periods should not be punished. Because $Z^2(HL)$ enters (3) negatively, its optimal value is zero when (4) is slack. There is no need to punish after HL when the Agent already has enough incentive to work after $(1, G)$. It is inefficient to punish after LH as long as (4) is binding and $\rho < 1$. Starting with any maximum-effort inducing Z^2 that has $Z^2(LH) > 0$, the Principal can relax (4) and keep the objective function and the left-hand-side of (3) constant by lowering $Z^2(LH)$ by $\varepsilon > 0$ and raising $Z^2(LL)$ by $\varepsilon p/(1-p)$. Such a change reuses the first-period punishment in the second period. As $Z^2(LH)$ declines and $Z^2(LL)$ increases, (4) becomes slack. Because both the objective function and the constraint (3) are linear in $(pZ^2(LH) + (1-p) Z^2(LL))$, any combination of $Z(LL)$ and $Z(HL)$ that satisfies (3) while keeping (4) slack is a solution. The strategy in Fuchs (2007), which punishes only after LL, corresponds to setting $\eta = 0$.

To see that $\widehat{Z}^2$ induces maximum effort when $\rho \leq 1 - \rho$, note that

$$\widehat{Z}^2(HL) - \widehat{Z}^2(HH) < \widehat{Z}^2(LL) - \widehat{Z}^2(LH).$$

That is, the punishment for a second-period L signal is more severe if the first-period signal is also L. By Assumption 2 the Agent's posterior belief that $y_1 = L$ is the lowest when $(e_1, s_1) = (1, G)$. As working is optimal after $(1, G)$ under $\widehat{Z}^2$ when $\rho \leq 1 - \delta$, it must also be optimal after any $(e_1, s_1) \in \{(0, G), (0, B), (1, B)\}$. This implies that working is always optimal in the second period. But, by (3), working in both periods is better than working only in the second.

The expected money-burning cost under $\widehat{Z}^2$ is $C(\widehat{Z}^2) = (1-p)c/(p-q)$. Note that $C(\widehat{Z}^2)$ is the same as the one-period money-burning cost $C(\overline{Z}^1)$. In this case the recycling of the first-period punishment lets the Principal motivate the Agent in the second period for free. Fuchs (2007) shows that when $\rho = 0$, the money-burning cost under the optimal contract is independent of T. We show that the result still holds when ρ is sufficiently low.

We now turn to the case where $\rho > 1 - \delta$. The optimization problem (5) in this case has a unique solution

$$\overline{Z}^2(y^2) = \begin{cases} \frac{c}{p-q}\left(\frac{1}{1-p} + \delta + \rho - 1\right) & \text{if } y^2 = LL, \\ 0 & \text{if } y^2 = LH, \\ \frac{c}{p-q}\left(\delta + \rho - 1\right) & \text{if } y^2 = HL, \\ 0 & \text{if } y^2 = HH. \end{cases}$$

Both (3) and (4) are binding at $\overline{Z}^2$. An interesting feature of $\overline{Z}^2$ is that it is asymmetric. Under it, an Agent with a high performance in the first period and a low performance in the second period is punished ($\overline{Z}^2(HL) > 0$), but an Agent with a low performance in the first period and a high performance in the second is not ($\overline{Z}^2(LH) = 0$). By contrast, under $\widehat{Z}^2$ the Agent is not punished in the first situation and slightly punished in the second ($\eta \leq \overline{\eta}$). In both $\widehat{Z}^2$ and $\overline{Z}^2$, the light or zero punishment after LH is driven by the need to reuse the first-period punishment in the second period. However, when $\rho > 1 - \delta$, the likelihood of $y_1 = L$ after $(1, G)$ is so heavily discounted that if the Principal tries to meet the second-period constraint after $(1, G)$ by punishing only after LL, he will need to set $Z^2(LL)$ at a level that makes the first-period constraint slack. But when (3) is slack, the Principal can reduce the money-burning cost by raising $Z^2(HL)$ and reducing $Z^2(LL)$.

The argument that shows $\widehat{Z}^2$ induces maximum effort when $\rho \leq 1 - \delta$ also shows that $\overline{Z}^2$ induces maximum effort when $\rho > 1 - \delta$. The expected money-burning cost under $\overline{Z}^2$ is

$$C(\overline{Z}^2) = \frac{(1-p)c}{p-q}\,(\delta + \rho)\,.$$

Note that in this case the money-burning cost strictly increases in ρ and is higher than the one-period money-burning cost. When $\rho = 1$, $C(\overline{Z}^2)$ is equal to $(1+\delta)\,(1-p)c/(p-q)$, the money-burning cost when the two periods are treated separately. There is no gain to link the punishment decisions across the two periods when the Agent can observe the Principal's signal.

When is Maximum Effort Optimal?

As we saw in the last section, the money-burning cost is independent of T when $\rho \leq 1 - \delta$. The optimal contract in that case must induce either maximum or no effort. In this section we show that when $\rho > 1 - \delta$, the Principal may prefer the Agent to work only some of the time.

We focus on the case where the Agent is induced to work in period 1 and after $(1, B)$ in period 2.[8] In this case the punishment strategy Z^2 must satisfy the second-period incentive-compatibility constraint:

$$\pi(H|1,B)[Z(HL) - Z(HH)] + \pi(L|1,B)[Z(LL) - Z(LH)] \geq \frac{\delta c}{p-q}. \qquad (6)$$

[8]As we saw in the last section, the optimal punishment strategy that induces the Agent to work in period 1 and in period 2 after $(1, G)$ would also induce the Agent to work after $(1, B)$. Hence, not requiring the Agent to work in period 2 after $(1, B)$ will not lower the money-burning cost.

Let us assume for now that (6) is binding and that

$$Z(HL) - Z(HH) \le Z(LL) - Z(LH). \tag{7}$$

Then, the Agent's best response in period 2 would be to shirk after $(1, G)$. Suppose, in addition, $\pi(L|1, B) < \min(\pi(L|0, G), \pi(L|0, B))$ so that the Agent's best response in period 2 would be to work if he has shirked in period 1. To induce the Agent to work in period 1, Z^2 must satisfy the first-period incentive-compatibility constraint

$$\begin{aligned} &p[Z(LH) - Z(HH)] + (1-p)[Z(LL) - Z(HL)] \ge \\ &\frac{[1-\delta\pi(G|1)]c}{p-q} + \pi(H, G|1)[Z(HL) - Z(HH)] + \pi(L, G|1)[Z(LL) - Z(LH)].^9 \end{aligned} \tag{8}$$

The Principal's problem is to minimize the money-burning cost

$$[p\pi(L, B|1) + q\pi(L, G|1)]Z(LH) + [(1-p)\pi(L, B|1) + (1-q)\pi(L, G|1)]Z(LL) +$$

$$[p\pi(H, B|1) + q\pi(H, G|1)]Z(HH) + [(1-p)\pi(H, B|1) + (1-q)\pi(H, G|1)]Z(HL)$$

subject to (6) and (8). The standard Kuhn-Tucker method yields a unique solution Z^{2*} with

$$Z^{2*}(y^2) = \begin{cases} \frac{c}{p-q}\left(1 - \delta\pi(G|1) - \frac{\delta\pi(L,B|1)}{\pi(L|1,B)}\right) & \text{if } y^2 = LH, \\ \frac{c}{p-q}\left(1 - \delta\pi(G|1) + \frac{\delta(1-\pi(L,B|1))}{\pi(L|1,B)}\right) & \text{if } y^2 = LL, \\ 0 & \text{if } y^2 = HH \text{ or } HL.^{10} \end{cases}$$

[9] Working at $t = 1$ increases the chance of G, in which case, the agent would save the effort cost c in the second period. The extra term on the right hand side of the constraint, compared to (3), represents this extra gain.

[10] It is straightforward to verify that (8) is binding and (7) holds.

Because the Principal does not require the Agent to work in period 2 after G, there is no need to punish the Agent after HL. Instead, the agent will be punished after LH. An Agent who has shirked in period 1 always works in period 2, whereas an Agent who has worked in period 1 works in period 2 only after B. As a result, the Principal is more likely to observe H in period 2 when the Agent has shirked in period 1. It is, therefore, more efficient to motivate the Agent in the first period through $Z\,(LH)$ than $Z\,(HL)$.

The expected money-burning cost of Z^{2*} is

$$\frac{(1-p)c}{p-q}\left[1-\delta\pi(G|1)+\frac{\delta\pi(L,G|1)(1-q)}{\pi(L|1,B)(1-p)}\right]. \tag{9}$$

When the signals are almost independent (i.e., $\rho\sim 0$), the surplus under Z^{2*} is approximately equal to

$$(1+\delta)\,(1-c)+\delta\pi(G|1)(c-1)-\frac{(1-p)c}{p-q},$$

which is less than

$$(1+\delta)\,(1-c)-\frac{(1-p)c}{p-q},$$

the surplus under $\overline{Z}^2$, the optimal maximum-effort contract when $\rho>1-\delta$, because $c<1$. When the signals are almost perfectly correlated (i.e., $\rho\sim 1$), the surplus under Z^{2*} is approximately equal to

$$(1+\pi\,(B|1)\,\delta)\,(1-c)-\frac{(1-p)c}{p-q}[1-\delta\pi(G|1)],$$

whereas the total surplus under $\overline{Z}^2$ is approximately equal to

$$(1+\delta)\,(1-c)-\frac{(1-p)c\,(1+\delta)}{p-q}.$$

$\overline{Z}^2$ would generate a higher surplus when c and $\pi(B|1)$ are small, but Z^{2*} would generate a higher surplus when c and $\pi(B|1)$ are large. To summarize, inducing maximum effort tends to be optimal when ρ is low or c is small. But the Principal may let the Agent shirk after $(1, G)$ if such an event is unlikely and c is high.

4 Optimal T-period Contract

In this section we derive the optimal maximum-effort contract for $T > 2$. For any $y^T \in Y^T$, let y^T_{-t} denote the Principal's signals in periods other than t, and y^T_t denote the t-th component of y^T. Consider an Agent who has chosen e^{t-1} and observed s^{t-1} in the first $t-1$ periods, and who is planning to choose $e_k = 1$ in all future periods $k > t$ (if there is any). His posterior belief that the Principal's evaluations in periods other than t is y^T_{-t} would be

$$\mu_t(y^T_{-t}|e^{t-1}, s^{t-1}) \equiv \prod_{k=1}^{t-1} \pi(y^T_k|e_k, s_k) \prod_{k=t+1}^{T} \pi(y^T_k|1).^{11}$$

His expected payoff for working in period t and all subsequent periods is

$$B + W - \sum_{y^T \in Y^T} \mu_t(y^T_{-t}|e^{t-1}, s^{t-1})\pi(y^T_t|1)Z^T(y^T) - \sum_{k=1}^{t-1} e_k \delta^{k-1} c - \sum_{k=t}^{T} \delta^{k-1} c.$$

His expected payoff for shirking in period t and working in all subsequent periods is

$$B + W - \sum_{y^T \in Y^T} \mu_t(y^T_{-t}|e^{t-1}, s^{t-1})\pi(y^T_t|0)Z^T(y^T) - \sum_{k=1}^{t-1} e_k \delta^{k-1} c - \sum_{k=t+1}^{T} \delta^{k-1} c.$$

[11] If $t = T$, then the second product term equals 1.

The Agent, therefore, prefers the former to the latter if

$$\sum_{y^T \in Y^T} \mu_t(y_{-t}^T | e^{t-1}, s^{t-1}) I(y_t) Z^T(y^T) \geq \frac{\delta^{t-1} c}{p-q}, \tag{10}$$

where

$$I(y_t) = \begin{cases} -1 & if \quad y_t = H, \\ 1 & if \quad y_t = L. \end{cases}$$

Any contract that induces maximum effort must satisfy (10) for all $t = 1, ..., T$, $e^{t-1} \in \{1,0\}^{t-1}$, and $s^{t-1} \in \{G, B\}^{t-1}$. The following lemma establishes the converse.

Lemma 1. *Z^T induces maximum effort if (10) holds for all $t = 1, ..., T$, $e^{t-1} \in \{1,0\}^{t-1}$, and $s^{t-1} \in \{G, B\}^{t-1}$.*

Proof. It is optimal for the Agent to work in period T after history (e^{T-1}, s^{T-1}) if (10) holds for $t = T$. Suppose starting from period $t+1$ it is optimal for the Agent to work in all remaining periods regardless of his effort choices and signals during the first t periods. Then, it would be optimal for the Agent to work in period t after history of (e^{t-1}, s^{t-1}) if (10) holds. The lemma is true by induction. □

The case of $\rho \leq 1 - \delta$

We showed in the last section that when $T = 2$ the optimal maximum-effort contract in Fuchs (2007), which punishes only when the evaluation is poor in every period, remains optimal so long as $\rho \leq 1 - \delta$. The following proposition extends the result to all T.

Proposition 2. *Let* $\mathbf{L}^T$ *denote a* T*-vector of* L*'s. When* $T > 1$ *and* $\rho \le 1 - \delta$*, it is efficient to induce maximum effort through the punishment strategy*

$$\widehat{Z}^T(y^T) = \begin{cases} \left(\dfrac{c}{p-q}\right)\dfrac{1}{(1-p)^{T-1}} & \text{if } y^T = \mathbf{L}^T, \\ 0 & \text{if } y^T \neq \mathbf{L}^T, \end{cases}$$

with expected money-burning cost $C(\widehat{Z}^T) = (1-p)c/(p-q)$*.*

Proof. By Lemma 1, $\widehat{Z}^T$ induces maximum effort if at any t and for any history (e^{t-1}, s^{t-1}), it satisfies the Agent's incentive constraint (10), which can be written as

$$\left[\prod_{k=1}^{t-1} \pi(L|e_k, s_k)\right](1-p)^{T-t}\widehat{Z}^T(L^T) \ge \frac{\delta^{t-1}c}{p-q}. \tag{11}$$

Under Assumption 2 and the condition $\rho \le 1-\delta$, $\pi(L|e_k, s_k) \ge \delta(1-p)$ for all $e_k \in \{0,1\}$ and $s_k \in \{G, B\}$. Thus, we have

$$\left[\prod_{k=1}^{t-1} \pi(L|e_k, s_k)\right](1-p)^{T-t}\widehat{Z}^T(L^T) \ge \delta^{t-1}(1-p)^{T-1}\widehat{Z}^T(L^T) \ge \frac{\delta^{t-1}c}{p-q},$$

indicating the Agent has no incentives to shirk at any t and for any history (e^{t-1}, s^{t-1}).

In this case the expected money-burning loss is equal to the minimum money-burning loss to induce effort in period 1. Because any contract that induces maximum effort must induce effort in the period 1, $\widehat{Z}^T$ must have the lowest money-burning cost among maximum-effort contracts. □

As we saw in the $T = 2$ case, $\widehat{Z}^T$ is not the unique efficient maximum-effort contract.[12] Proposition 2 only shows that $\widehat{Z}^T$ is the most efficient way to induce

[12]See the discussion in the next section.

maximum effort. Fuchs (2007) shows that $\widehat{Z}^T$, in fact, generates the greatest surplus among all contracts when T is large. A sketch of his argument is as follows. The per-period money-burning cost under $\widehat{Z}^T$ converges to 0 as T tends to infinity. Hence, when T becomes large, the surplus under $\widehat{Z}^T$ must be strictly positive and, therefore, superior to a contract that induces no effort.[13] If it is worthwhile to induce any effort, then it would be worthwhile to induce effort in the first period. But any contract that induces effort in the first period must incur at least $C(\widehat{Z}^T)$.

The case of $\rho > 1 - \delta$

Recall that when $T = 2$, the value of $\overline{Z}^2(HL)$ and $\overline{Z}^2(LL)$ are pinned down by the binding constraints (3) and (4). Setting $\overline{Z}^2(HH)$ and $\overline{Z}^2(LH)$ to zero and replacing the inequality signs with equality ones, we can rewrite these constraints as

$$(1-p)[\overline{Z}^2(LL) - \overline{Z}^2(HL)] = \frac{c}{p-q}, \tag{12}$$

$$\pi(H|1,G)\overline{Z}^2(HL) + \pi(L|1,G)\overline{Z}^2(LL) = \delta\overline{Z}^1(L), \tag{13}$$

where $\overline{Z}^1(L) = c/(p-q)$ is the optimal punishment after L in a one-period maximum-effort contract. Thus, $\overline{Z}^2(HL)$ and $\overline{Z}^2(LL)$ are chosen such that their difference is sufficient to induce the Agent to work in the first period, and that conditional on $(1, G)$ the effective punishment is equal to that of a one-period contract discounted by δ. This observation suggests we can derive an optimal T-period maximum-effort contract recursively.

[13]Recall that we assume that working is efficient when there is no need to burn money; i.e., $c < 1$.

Let $x \circ y_{-1}^T \equiv (x, y_2, ...y_T)$ denote the T-period history that starts with $x \in \{H, L\}$ and is followed by $y_{-1}^T \equiv (y_2, ..., y_T)$. Let $\mathbf{L}^{T-1}$ be a $(T-1)$ vector of L's. Set $\overline{Z}^1(H) = 0$ and $\overline{Z}^1(L) = c/(p-q)$. For $T \geq 2$, set $\overline{Z}^T\left(H \circ \mathbf{L}^{T-1}\right)$ and $\overline{Z}^T\left(L \circ \mathbf{L}^{T-1}\right)$ such that

$$(1-p)^{T-1}\left[\overline{Z}^T(L \circ \mathbf{L}^{T-1}) - \overline{Z}^T(H \circ \mathbf{L}^{T-1})\right] = \frac{c}{p-q}, \tag{14}$$

and

$$\pi\left(H|1, G\right)\overline{Z}^T(H \circ \mathbf{L}^{T-1}) + \pi\left(L|1, G\right)\overline{Z}^T(L \circ \mathbf{L}^{T-1}) = \delta\overline{Z}^{T-1}(\mathbf{L}^{T-1}). \tag{15}$$

For any $y^T \in \{H, T\}^T$ that contains an "H" signal after period 1, set $\overline{Z}^T\left(y^T\right)$ to $\delta\overline{Z}^{T-1}(y_{-1}^T)$.

This yields

$$\overline{Z}^T(y^T) \equiv \begin{cases} \delta\overline{Z}^{T-1}(\mathbf{L}^{T-1}) + \dfrac{\pi(H|1,G)}{(1-p)^{T-1}}\left(\dfrac{c}{p-q}\right) & \text{if } y_1 = L \text{ and } y_{-1}^T = \mathbf{L}^{T-1}, \\ \delta\overline{Z}^{T-1}(\mathbf{L}^{T-1}) - \dfrac{\pi(L|1,G)}{(1-p)^{T-1}}\left(\dfrac{c}{p-q}\right) & \text{if } y_1 = H \text{ and } y_{-1}^T = \mathbf{L}^{T-1}, \\ \delta\overline{Z}^{T-1}(y_{-1}^T) & \text{if } y_{-1}^T \neq \mathbf{L}^{T-1}. \end{cases} \tag{16}$$

Note that $\overline{Z}^T$ depends only on the time the Principal last observes H. Let $\tilde{t}\left(y^T\right) \equiv \max\{t|y_t = H\}$ denote the last period H occurs in y^T. Solving $\overline{Z}^T$ recursively, we obtain

$$\overline{Z}^T(y^T) = \begin{cases} 0 & \text{if } y_T^T = H, \\ \dfrac{(1-p)c}{p-q}(\delta+\rho-1)\sum_{t=1}^{T-\tilde{t}(y^T)}\dfrac{\delta^{T-1-t}}{(1-p)^t} & \text{if } y^T \neq L^T, \\ \dfrac{(1-p)c}{p-q}\left[\dfrac{1}{(1-p)^T} + (\delta+\rho-1)\sum_{t=1}^{T-1}\dfrac{\delta^{T-1-t}}{(1-p)^t}\right] & \text{if } y^T = L^T. \end{cases} \tag{17}$$

It is straightforward to check that $\overline{Z}^T$ is positive and strictly decreasing in $\tilde{t}\left(y^T\right)$ when $\delta + \rho - 1 > 0$.

Proposition 3. *When $\rho > 1 - \delta$, it is efficient to induce maximum effort through the punishment strategy $\overline{Z}^T$. The expected money-burning cost of $\overline{Z}^T$ is*

$$C\left(\overline{Z}^T\right) = \frac{(1-p)c}{p-q}\left[\delta^{T-1} + \rho\sum_{t=1}^{T-1}\delta^{t-1}\right]. \tag{18}$$

Proof. We prove the Proposition in two steps. First, we establish a lower bound on the money-burning cost of any maximum-effort contracts. Note that any maximum-effort inducing Z^T must satisfy (10) for $t = 1$, which can be written as

$$\sum_{y_{-1}^T \in Y^{T-1}} \mu_1\left(y_{-1}^T | e^0, s^0\right)\left[Z^T(L \circ y_{-1}^T) - Z^T(H \circ y_{-1}^T)\right] \geq \frac{c}{p-q}. \tag{19}$$

Given Z^T, define for all $y_{-1}^T \in \{H, L\}^{T-1}$

$$Z^{T-1}(y_{-1}^T) \equiv \frac{1}{\delta}[\pi(H|1,G)Z^T(H \circ y_{-1}^T) + \pi(L|1,G)Z^T(L \circ y_{-1}^T)]. \tag{20}$$

An Agent who has worked and observed G in period 1 is effectively facing Z^{T-1} from period 2 onward.

Because Z^T, by supposition, induces maximum effort, it must be a best response for the Agent to work in all subsequent periods after working and observing G in the first. Hence, Z^{T-1} must induce maximum effort in a $(T-1)$-period contracting game. Using (19) and (20), we have

$$
\begin{aligned}
C(Z^T) &= \sum_{y^T \in Y^T} \left[\prod_{k=1}^{T} \pi(y_k|1) \right] Z^T(y^T) \qquad (21) \\
&= \sum_{y_{-1}^T \in Y^{T-1}} \mu_1 \left(y_{-1}^T | e^0, s^0 \right) \left[p Z^T(H \circ y_{-1}^T) + (1-p) Z^T(L \circ y_{-1}^T) \right] \\
&= \sum_{y_{-1}^T \in Y^{T-1}} \mu_1(y_{-1}^T | e^0, s^0) \{ \delta Z^{T-1}(y_{-1}^T) + [(1-p) - \pi(L|1,G)][Z^T(L \circ y_{-1}^T) - Z^T(H \circ y_{-1}^T)] \} \\
&= \delta C(Z^{T-1}) + \rho\,(1-p) \sum_{y_{-1}^T \in Y^{T-1}} \mu_1 \left(y_{-1}^T | e^0, s^0 \right) \left[Z^T(L \circ y_{-1}^T) - Z^T(H \circ y_{-1}^T) \right] \\
&\geq \delta C(Z^{T-1}) + \frac{\rho\,(1-p)\,c}{p-q}.
\end{aligned}
$$

The above inequality provides a lower bound on the money-burning cost. The minimum money-burning cost when $T = 1$ is $(1-p)c/(p-q)$ (Proposition 1). Applying this relation recursively, we find that the money-burning cost in inducing maximum effort in a T-period contracting game cannot be lower than

$$
\frac{(1-p)c}{p-q} \left[\delta^{T-1} + \rho \sum_{t=1}^{T-1} \delta^{t-1} \right]. \qquad (22)
$$

Next, we show that $\overline{Z}^T$ induces maximum effort and has a money-burning cost equal to the lower bound in (22). By Proposition 1, $\overline{Z}^1$ induces effort when $T = 1$. Now suppose $\overline{Z}^{T-1}$ induces maximum effort in the $(T-1)$-period contracting game for some $T \geq 2$. Consider the T-period contracting game. Under $\overline{Z}^T$, the incentive constraint for $t = 1$ as in (19) holds because, by construction,

$$
\overline{Z}^T(L \circ \mathbf{L}^{T-1}) - \overline{Z}^T(H \circ \mathbf{L}^{T-1}) = \frac{c}{(1-p)^{T-1}(p-q)},
$$

and $\overline{Z}^T(L \circ y_{-1}^T) = \overline{Z}^T(H \circ y_{-1}^T)$ for all $y_{-1}^T \neq \mathbf{L}^{T-1}$. Note that the punishment strategy $\overline{Z}^T$ satisfies the condition that for all $y_{-1}^T \in \{H, L\}^{T-1}$,

$$\pi(H|1, G)\,\overline{Z}^T(H \circ y_{-1}^T) + \pi(L|1, G)\,\overline{Z}^T(L \circ y_{-1}^T) = \delta\overline{Z}^{T-1}(y_{-1}^T).$$[14]

The Agent is therefore effectively facing the punishment strategy $\overline{Z}^{T-1}$ in period 2 after working and observing G in period 1. Let $IC(e^{t-1}, s^{t-1})$ denote the incentive-compatibility constraint (10) and $IC(1 \circ e_{-1}^T, G \circ s_{-1}^T)$ the same constraint conditional on observing $(1 \circ e_{-1}^T, G \circ s_{-1}^T)$. Because $\overline{Z}^{T-1}$ induces maximum effort, $IC(1 \circ e_{-1}^T, G \circ s_{-1}^T)$ must hold for all $\left(e_{-1}^T, s_{-1}^T\right)$. By Assumption 2, $\pi(L|1, G) \leq \pi(L|e_1, s_1)$ for all (e_1, s_1) and, thus,

$$\begin{aligned}
& \sum_{y^T \in Y^T} \left[\mu_t\left(y_{-t}^T | e^{t-1}, s^{t-1}\right) - \mu_t\left(y_{-t}^T | 1 \circ e_{-1}^{t-1}, G \circ s_{-1}^{t-1}\right)\right] I(y_t)\,\overline{Z}^T\left(y^T\right) \\
= & \left[\prod_{k=2}^{t-1} \pi(L|e_k, s_k)\right] \pi(L|1)^{T-t+1} \left[\pi(L|e_1, s_1) - \pi(L|1, G)\right] \left[\overline{Z}^T\left(\mathbf{L}^T\right) - \overline{Z}^T\left(H \circ \mathbf{L}^{T-1}\right)\right] \\
\geq & \ 0.
\end{aligned}$$

This implies that for all $t \geq 2$ and for all (e^{t-1}, s^{t-1}), the left-hand side of $IC(e^{t-1}, s^{t-1})$ is greater than the left-hand side of $IC(1 \circ e_{-1}^{t-1}, G \circ s_{-1}^{t-1})$. As $IC(1 \circ e_{-1}^{t-1}, G \circ s_{-1}^{t-1})$ holds, $IC\left(e^{t-1}, s^{t-1}\right)$ also holds. Hence, by Lemma 1, $\overline{Z}^T$ induces maximum effort. Because $\overline{Z}^T$ satisfies the incentive constraint (19) with equality for all T, it has a money-burning cost equal to the lower bound in (22). □

We saw that when $T = 2$, the optimal punishment depends only on the last time the Principal's evaluation is H (i.e., $\overline{Z}^2(LH) = \overline{Z}^2(HH)$). Proposition 3 shows that

[14]This follows from condition (15) and that $\overline{Z}^T(y^T) = \delta\overline{Z}^{T-1}(y_{-1}^T)$ for all $y_{-1}^T \neq \mathbf{L}^{T-1}$.

this property holds for all T. Once the Agent obtains a high evaluation in period t, all the low evaluations from periods 1 to $t-1$ are "forgiven." The punishment under $\overline{Z}^T$ is strictly lower when the last H occurs later. There is no punishment if the Agent performs well in the last period. Given the same evaluations, the level of punishment decreases with the correlation coefficient ρ. As ρ tends to $1-\delta$ (from above), $\overline{Z}^T$ converges to $\widehat{Z}^T$. Hence, the optimal maximum-effort contract we develop includes that of Fuchs (2007) as a special case.

Corollary 1. *When $\rho > 1-\delta$, the minimum money-burning cost $C(\overline{Z}^T)$ increases in the correlation coefficient ρ, converges to $C(\overline{Z}^1)$ as $\rho \to 1-\delta$, and converges to $C(\overline{Z}^1)(1-\delta^T)/(1-\delta)$ as $\rho \to 1$. For any fixed $\rho > 1-\delta$, the per-period money burning cost $(1-\delta)C(\overline{Z}^T)/\left(1-\delta^T\right)$ decreases in T and converges to $\rho(1-p)c/(p-q)$ as T tends to infinity.*

Corollary 1 generalizes the result in the $T=2$ case. When $\rho > 1-\delta$, the money-burning loss is no longer independent of T and ρ. Having a longer review horizon reduces the per-period money-burning cost, but some inefficiency would remain even when T goes to infinity.[15] Thus, it may not be optimal to induce maximum effort even when T is very large.

When $T=2$, the only way to motivate the Agent to work in the first period is through the difference between $\overline{Z}^2(LL)$ and $\overline{Z}^2(HL)$, as any contract that involves a

[15]In a two-period tournament model, O'Keeffe et al. (1984) have also shown that learning by the Agent can hurt the Principal.

strictly positive punishment after LH is inefficient. Hence, $\overline{Z}^2$ is the unique efficient maximum-effort contract. When $T = 3$, under our construction the incentive to work in the first period is entirely provided through the difference between $\overline{Z}^3(LLL)$ and $\overline{Z}^2(HLL)$, but the incentive could also be provided in part through the difference in punishment after of LHL and HHL.[16] Hence, $\overline{Z}^T$ is not uniquely efficient when $T > 2$. Nevertheless, any efficient punishment strategies must differentiate between H signals in different periods.

Proposition 4. *Any maximum-effort-inducing punishment strategy Z^T that depends only on the total number of H signals is inefficient.*

Proof. Consider any Z^T that induces maximum effort for some $T \geq 2$. Define Z^{T-1} according to (20). Following the arguments in Proposition 3, Z^{T-1} must also induce maximum effort, and

$$C\left(Z^T\right) \geq \delta C\left(Z^{T-1}\right) + \frac{\rho(1-p)c}{p-q}.$$

In Proposition 3, we have already shown $\overline{Z}^T$ is efficient and

$$C(\overline{Z}^T) = \delta C(\overline{Z}^{T-1}) + \frac{\rho(1-p)c}{p-q}.$$

It follows that $C(Z^T) > C(\overline{Z}^T)$ if $C(Z^{T-1}) > C(\overline{Z}^{T-1})$. Hence, Z^T can be efficient only if Z^{T-1} is efficient.

[16] For example, reduce $\overline{Z}^3(LLL)$ by ε and $\overline{Z}^3(HHL)$ by $\varepsilon\frac{1-p}{p}\frac{\pi(L|1,G)}{\pi(H|1,G)}$; increase $\overline{Z}^3(LHL)$ by $\varepsilon\frac{1-p}{p}$ and $\overline{Z}^3(HLL)$ by $\varepsilon\frac{\pi(L|1,G)}{\pi(H|1,G)}$. The resulting punishment strategy will also induce maximum effort efficiently when $\varepsilon > 0$ is sufficiently small.

But in Proposition 1 we have already seen that $\overline{Z}^1$ (with $\overline{Z}^1(H) = 0$) is uniquely efficient when $T = 1$. It follows that any Z^T with $Z^T(y^T) > 0$ for some y^T with $y_T = H$ must be inefficient. However, if Z^T depends only on the total number of H signals (and not on when they occur), the only way to ensure that $Z^T(y^T) = 0$ for any y^T that ends with H would be to set $Z^T(y^T) = 0$ for all $y^T \neq \mathbf{L}^T$. But we have already seen that $\widehat{Z}^T$ is inefficient in inducing maximum effort when $\rho > 1 - \delta$. □

5 Communication

Many firms provide workers performance feedbacks and ask their workers to evaluate their own performances. Proposition 5 says that when $1 - q \geq \pi(L|1, B)$, communication between the Principal and Agent does not reduce inefficiency.[17] Because $p > q$, the condition that $1 - q \geq \pi(L|1, B)$ is satisfied when the Agent's signal is not too informative.

Proposition 5. $\overline{Z}^T$ *is optimal among all punishment strategies with communication that induces maximum effort when* $\rho > 1 - \delta$ *and* $1 - q \geq \pi(L|1, B)$.

Proof. See the appendix. □

We can illustrate the intuition behind the proposition with the one-period case. Consider the case where $\pi(L|0, G) = \pi(L|0, B) = 1 - q < \pi(L|1, B)$. Suppose after the

[17]Obviously, such communication may serve purposes not modeled in this article.

[Insert Figure 1 here]

Agent has observed s_1, the Principal offers him a choice between $\overline{Z}^1$ and $\tilde{Z}^1$, where

$$\overline{Z}^1(H) = 0; \quad \overline{Z}^1(L) = \frac{c}{p-q};$$

and

$$\tilde{Z}^1(H) = \frac{(1-q)c}{q(p-q)}; \quad \tilde{Z}^1(L) = 0.$$

As shown in panel a of Figure 1, an Agent who has shirked would be indifferent between $\overline{Z}^1$ and $\tilde{Z}^1$. Hence, offering this additional choice would not benefit a shirking Agent. But an Agent who has worked and observed B would be strictly better off choosing $\tilde{Z}^1$. Offering this choice, therefore, simultaneously lowers the efficiency loss and provides a greater incentive to work. Intuitively, in this case working provides the Agent a very informative signal about y_1 that the Principal could exploit to punish a shirking Agent.

The same approach would not work when $1 - q \geq \pi(L|1, B)$. (Here, we do not assume that $\pi(L|0, G) = \pi(L|0, B)$.) Suppose the Principal again allows the Agent to choose from a set of punishment agreements, and in equilibrium it is optimal for the Agent to work and choose $Z^1_{s_1}$ after observing $s_1 \in \{G, B\}$. Incentive-compatibility requires that

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|1,G\right) Z_B^1\left(y_1\right) \geq \sum_{y_1\in\{H,L\}} \pi\left(y_1|1,G\right) Z_G^1\left(y_1\right); \tag{23}$$

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|1,B\right) Z_B^1\left(y_1\right) \leq \sum_{y_1\in\{H,L\}} \pi\left(y_1|1,B\right) Z_G^1\left(y_1\right). \tag{24}$$

Combining (23) and (24), we have

$$Z_G^1\left(H\right) \leq Z_B^1\left(H\right) \text{ and } Z_G^1\left(L\right) \geq Z_B^1\left(L\right). \tag{25}$$

Furthermore, there must exist some $\left(Q\left(H\right), Q\left(L\right)\right) \geq 0$ such that

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|1,G\right) Z_G^1\left(y_1\right) = \sum_{y_1\in\{H,L\}} \pi\left(y_1|1,G\right) Q\left(y_1\right); \tag{26}$$

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|1,B\right) Z_B^1\left(y_1\right) = \sum_{y_1\in\{H,L\}} \pi\left(y_1|1,B\right) Q\left(y_1\right). \tag{27}$$

See panel b of Figure 1. As $1-q \geq \pi(L|1,B)$, it follows from (27) and (25) that

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|0\right) Q\left(y_1\right) > \sum_{y_1\in\{H,L\}} \pi\left(y_1|0\right) Z_B^1\left(y_1\right). \tag{28}$$

Because the Agent prefers working to shirking and reporting B, we have

$$\sum_{y_1\in\{H,L\}} \pi\left(y_1|0\right) Z_B^1\left(y_1\right) - \sum_{y_1\in\{H,L\},s_1\in\{G,B\}} \pi\left(y_1,s_1|1\right) Z_{s_1}^1\left(y_1\right) \geq c. \tag{29}$$

Substituting (26), (27), and (28) into (29), we have

$$Q\left(L\right) - Q\left(H\right) \geq \frac{c}{p-q}. \tag{30}$$

It follows that

$$\sum_{y_1\in\{H,L\},s_1\in\{G,B\}} \pi\left(y_1,s_1|1\right) Z_{s_1}^1\left(y_1\right) = \sum_{y_1\in\{H,L\}} \pi\left(y_1|1\right) Q\left(y_1\right) \geq \frac{(1-p)c}{p-q}. \tag{31}$$

Intuitively, when $1 - q \geq \pi(L|1, B)$, any punishment scheme that benefits an Agent who has worked and observed B compared to $\overline{Z}^1$ would benefit a shirking Agent even more. Hence, letting the Agent communicates his signal will not improve the efficiency of a maximum-effort contract.

Now suppose before the contract is signed the Principal announces that he will send the Agent a message $d_1 \in D$ with probability $\gamma(d_1|y_1)$ before letting the Agent choose from a menu of punishment agreements.[18] D can be any finite message space. Let $Z^1_{s_1,d_1}$ denote the punishment agreement that the Agent chooses in equilibrium when his own signal is s_1 and the Principal's message is d_1. Assume that the contract is designed so that it is optimal for the Agent to work.

Let $\pi(.|e_1, s_1, d_1)$ denote the Agent's posterior belief conditional on (e_1, s_1, d_1). Incentive compatibility requires that for each $d_1 \in D$

$$\sum_{y_1 \in \{H,L\}} \pi(y_1|1, G, d_1) Z^1_{B,d_1}(y_1) \geq \sum_{y_1 \in \{H,L\}} \pi(y_1|1, G, d_1) Z^1_{G,d_1}(y_1); \tag{32}$$

$$\sum_{y_1 \in \{H,L\}} \pi(y_1|1, B, d_1) Z^1_{B,d_1,}(y_1) \leq \sum_{y_1 \in \{H,L\}} \pi(y_1|1, B, d_1) Z^1_{G,d_1}(y_1). \tag{33}$$

For each $(s_1, y_1) \in \{G, B\} \times \{H, L\}$, with a slight abuse of notation, let

$$Z^1_{s_1}(y_1) \equiv \sum \gamma(d_1|y_1) Z^1_{s_1,d_1}(y_1)$$

denote the average punishment the Agent with signal s_1 will receive in equilibrium when the Principal's signal is y_1. Multiplying both sides of (32) by $\pi(d_1|1, G)$ (the

[18] As ex post the Principal is indifferent to the amount of money burnt, he will have no incentive to deviate from γ.

probability that d_1 will be sent conditional on $e_1 = 1$ and $s_1 = G$) and summing over d_1, we obtain (23). By the same method, we obtain (24). Equations (25)-(28) then follow. Lastly, note that an Agent who shirks and chooses $Z^1_{B,d_1}(y_1)$ when the Principal sends the message d_1 will receive an average penalty of

$$\sum_{y_1 \in \{H,L\}} \pi(y_1|0) \left(\sum_{d_1 \in D} \gamma(d_1|y_1) Z^1_{B,d_1}(y_1) \right) = \sum_{y_1 \in \{H,L\}} \pi(y_1|0) Z^1_B(y_1).$$

Thus, (29) and, therefore, (30) and (31) must hold if working is optimal for the Agent. This establishes that when $T = 1$ communication does not improve efficiency when $1 - q \geq \pi(L|1, B)$. The proof in the Appendix uses a recursive argument similar to the one in the proof of Proposition 3 to show that the result is true for all $T \geq 1$

6 Conclusion

Fuchs (2007) shows that an employer can reduce the efficiency loss of an efficiency-wage contract by adopting a long review horizon when the worker has no information about the employer's evaluation of his performance. In this article we extend his analysis to the more general case where the worker's self-evaluation is correlated with the employer's evaluation.

We show that a contract that induces maximum effort efficiently in this environment has two features. First, to prevent a worker with good self-evaluations early on from shirking in the subsequent periods, the contract should punish a worker whose performance deteriorates over time. Second, because punishment is costly, the contract should let a worker make up for his poor evaluations in the past with better future ones. As a result, the optimal incentives must be asymmetric. Any contract that depends only on the total number of high evaluations would be inefficient.

The need to punish a worker who has performed well up to a certain period reduces his incentives to work in those periods. As a result, the efficiency loss of an efficient-wage contract is increasing in the correlation between evaluations of the worker and the employer. As the correlation diminishes, both the contract and the efficiency loss converge to those in Fuchs (2007). However, as the correlation goes to one, the efficiency loss converges to the efficiency loss of T one-period efficiency-wage contracts.

In Fuchs (2007) it is always optimal to induce maximum effort. By contrast, we show in the two-period case when the surplus of effort is small and the correlation between evaluations is high, it could be better for the employer to let a worker who has worked and received a good self-evaluation in period 1 shirk in period 2. We also show that communication between the employer and the worker could reduce efficiency loss when the correlation between evaluations is sufficiently high. These results suggest that the contract that we identify in this paper, which induces maximum effort at the lowest cost and involves no communication, maximizes total surplus only when effort is highly productive and the worker does not knows too much about the employer's evaluations. Future research should investigate whether the idea of reducing efficiency loss through rewarding improving performance applies more generally.

Appendix

Suppose in each period t after the realization of the signals, the Principal sends a message $d_t \in D$ to the Agent with probability $\gamma_t(d_t|y_t)$, and, after receiving d_t, the Agent sends the Principal a message m_t from a message set $M_t \subseteq \{G, B\}$ at the end of each period t after the realization of s_t. D can be any finite message space. We assume that the Agent knows $\gamma = (\gamma_1, ..., \gamma_T)$. The Agent's history at date t for $t > 1$ now includes the messages he received from the Principal, the messages he sent, his effort choices, and the private evaluations he observed in the previous $t-1$ periods. A message strategy is a vector $\phi \equiv (\phi_1, ..., \phi_T)$, where ϕ_t is a function that maps each feasible period-t history to a message in M_t. At the end of period T, the Principal will have observed T messages $m^T \equiv (m_1, ..., m_T)$, in addition to his T private signals $y^T \equiv (y_1, \ldots, y_T,)$. A punishment strategy Z^T is then a function that maps each $\left(y^T, m^T\right)$ in $\{H, L\}^T \times \{M_t\}_{t=1}^T \times D^T$ to a real number in $[0, W]$. Let $v(Z^T, \sigma, \phi)$ denote the Agent's expected effort and punishment cost under (σ, ϕ). An Agent's strategy (σ^*, ϕ^*) is a best response against Z^T if, for all feasible strategies (σ, ϕ), the expected cost under (σ, ϕ) is higher than that under (σ^*, ϕ^*),

$$v(Z^T, \sigma, \phi) \geq v(Z^T, \sigma^*, \phi^*).$$

We say that (γ, Z^T) induces maximum effort if a best response of the Agent against (γ, Z^T) involves working in every period. We say that (γ, Z^T) involves no communication if D is a singleton and Z^T is independent of m^T.

Proof of Proposition 5.

We establish the Proposition through two lemmas.

Lemma 2. *Consider the minimization problem*

$$\min_{Q(H),Q(L)} \pi(L|1,B)Q(L) + \pi(H|1,B)Q(H)$$

such that

$$\begin{aligned}
\pi(H|1,G)Q(H) + \pi(L|1,G)Q(L) &\geq \lambda, \\
(q - \pi(H,B|1))Q(H) + (1 - q - \pi(L,B|1))Q(L) &\geq c + \pi(G|1)\lambda.
\end{aligned}$$

Suppose $1 - q > \pi(L|1,B)$*. The solution to this problem satisfies the equation*

$$\begin{aligned}
\pi(H|1,B)Q(H) + \pi(L|1,B)Q(L) &= \frac{(\pi(L|1,B) - \pi(L|1,G))c}{p - q} + \lambda, \\
Q(L) - Q(H) &= \frac{c}{p - q}.
\end{aligned}$$

Proof. Note that

$$\frac{1 - q - \pi(L,B|1)}{q - \pi(H,B|1)} > \frac{\pi(L|1,B)}{\pi(H|1,B)} > \frac{\pi(L|1,G)}{\pi(H|1,G)}.$$

(The first inequality follows from $1 - q > \pi(L|1,B)$.) It is straightforward to show that both constraints are binding at the optimal solution.

In this case, we have

$$\pi(H|1,G)Q(H) + \pi(L|1,G)Q(L) = \lambda,$$

$$[q-\pi(H,B|1)]Q(H)+[1-q-\pi(L,B|1)]Q(L)=c+\pi(G|1)\lambda.$$

Solving the equation system yields

$$Q(H)=\frac{-\pi(L|1,G)c}{p-q}+\lambda,\quad Q(L)=\frac{\pi(H|1,G)c}{p-q}+\lambda.$$

□

Lemma 3. *Suppose the minimum expected money-burning cost to induce maximum effort in a T-period contracting game is C^T. Then the minimum expected money-burning cost to induce maximum effort in a $T+1$-period game is*

$$\delta C^T+\frac{\rho(1-p)c}{p-q}.$$

Proof. Consider some $\left(\gamma, Z^T\right)$ that induces maximum effort. By the revelation principle we can assume $\left(\gamma, Z^T\right)$ induces the Agent to report truthfully. Define for $y_1\in\{H,L\}$, $\widehat{s}_1\in\{G,B\}$, and $d_1\in D$,

$$\widetilde{Q}(y_1,\widehat{s}_1,d_1)\equiv\sum_{y_{-1}^{T+1}}\sum_{s_{-1}^{T+1}}\sum_{d_{-1}^{T+1}}\left[\prod_{t=2}^{T+1}\pi(y_t,s_t|1)\gamma_t\left(d_t|y_t\right)\right]Z^{T+1}(y_1\circ y_{-1}^{T+1},\widehat{s}_1\circ s_{-1}^{T+1},d_1\circ d_{-1}^{T+1}).$$

Here y_{-1}^{T+1}, s_{-1}^{T+1}, and d_{-1}^{T+1} denote, respectively, the private signals of the Principal and the Agent, and the messages sent by the Principal in periods 2 to $T+1$. $\widetilde{Q}(y_1,\widehat{s}_1,d_1)$ is the expected amount of money burnt if in the first period the Principal's signal and message are y_1 and d_1, and the Agent reports $\widehat{s}_1$ in the first period and exerts effort and reports truthfully in all subsequent periods, i.e., $e_t=1$ and $\widehat{s}_t=s_t$ for $t=2,\dots,T+1$. For $y_1\in\{H,L\}$, $\widehat{s}_1\in\{G,B\}$, let

$$Q\left(y_1,\widehat{s}_1\right)=\sum\gamma_1\left(d_1|y_1\right)\widetilde{Q}(y_1,\widehat{s}_1,d_1)$$

denote the average $\widetilde{Q}$ over d_1.

Note that an Agent who has exerted effort, received a G signal and a message d_1, and reported truthfully in the first period is effectively facing the punishment strategy

$$\begin{aligned} Z^T\left(y_{-1}^{T+1}, \widehat{s}_{-1}^{T+1}, d_{-1}^{T+1}\right) &= \pi(H|1,G,d_1)Z^{T+1}(H \circ y_{-1}^{T+1}, G \circ \widehat{s}_{-1}^{T+1}, d_1 \circ d_{-1}^{T+1}) \\ &\quad + \pi(L|1,G,d_1)Z^{T+1}(L \circ y_{-1}^{T+1}, G \circ \widehat{s}_{-1}^{T+1}, d_1 \circ d_{-1}^{T+1}) \end{aligned} \quad (34)$$

from period two onwards. It follows that

$$\pi(H|1,G,d_1)\widetilde{Q}(H,G,d_1) + \pi(L|1,G)\widetilde{Q}(L,G,d_1) \geq \delta C^T. \quad (35)$$

Incentive-compatibility requires that at the end period 1 the Agent, conditional on $(e_1, s_1, d_1) = (1, G, d_1)$, prefers reporting G to reporting B in that period and exerting effort and reporting honestly in all subsequent periods. This requires that

$$\begin{aligned} &\pi(H|1,G,d_1)\widetilde{Q}(H,B,d_1) + \pi(L|1,G,d_1)\widetilde{Q}(L,B,d_1) \geq \\ &\pi(H|1,G,d_1)\widetilde{Q}(H,G,d_1) + \pi(L|1,G,d_1)\widetilde{Q}(L,G,d_1). \end{aligned} \quad (36)$$

Inequalities (35) and (36) jointly imply

$$\pi(H|1,G,d_1)\widetilde{Q}(H,B,d_1) + \pi(L|1,G,d_1)\widetilde{Q}(L,B,d_1) \geq \delta C^T \quad (37)$$

Multiplying both sides of (37) by $\pi(d_1|1,G)$ and summing over d_1, we have

$$\pi(H|1,G)Q(H,B) + \pi(L|1,G)Q(L,B) \geq \delta C^T \quad (38)$$

In period 1, the Agent must prefer the equilibrium strategy to the strategy of shirking and reporting $(1, B)$ in period 1, followed by working and reporting truthfully in future periods. This requires that

$$\begin{aligned}&[qQ(H,B)+(1-q)Q(L,B)]- \\ &[\pi(H,G|1)Q(H,G)+\pi(L,G|1)Q(L,G)+\pi(H,B|1)Q(H,B)+\pi(L,B|1)Q(L,B)]\geq c.\end{aligned} \tag{39}$$

Using (35) and rearranging terms, we have

$$\begin{aligned}&[q-\pi(H,B|1)]Q(H,B)+[1-q-\pi(L,B|1)]Q(L,B) \\ \geq\ & c+\pi(G|1)\delta C^T.\end{aligned} \tag{40}$$

With the two conditions, (37) and (40), it follows from Lemma 2 that

$$\pi(H|1,B)Q(H,B)+\pi(L|1,B)Q(L,B)\geq \delta C^T+\frac{[\pi(L|1,B)-\pi(L|1,G)]c}{p-q}. \tag{41}$$

Combining conditions (35) and (41) gives

$$\begin{aligned}C^{T+1}=&\sum_{y_1\in\{H,L\},\widehat{s}_1\in\{G,B\}}\pi(y_1,\widehat{s}_1|1)Q(y_1,\widehat{s}_1) \\ \geq&\delta C^T[\pi(B|1)+\pi(G|1)]+\frac{\pi(B|1)[\pi(L|1,B)-\pi(L|1,G)]c}{p-q} \\ =&\delta C^T+\frac{[\pi(L|1)-\pi(L|1,G)]c}{p-q} \\ =&\delta C^T+\frac{\rho(1-p)c}{p-q}.\end{aligned} \tag{42}$$

We have already shown in text that $C^1=(1-p)c/(p-q)$. Solving recursively, we have

$$C^T\geq\frac{(1-p)c}{p-q}\left[\delta^{T-1}+\rho\sum_{t=1}^{T-1}\delta^{t-1}\right]$$

which is the same as the minimum money-burning cost when no communication is allowed. □

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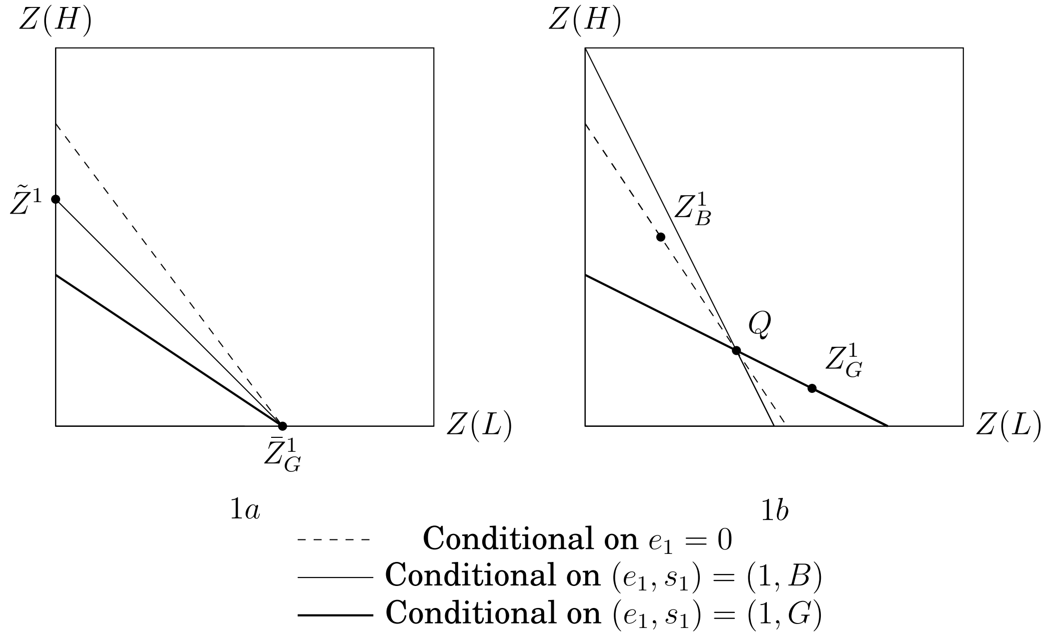

Figure 1: Indifference curves of the Agent