On the Non-Existence of Reputation Effects in Two-Person Infinitely-Repeated Games

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Abstract

Consider a two-person infinitely-repeated game in which one player is either a normal "rational" type or a "commitment" type that automatically plays a fixed repeated-game strategy. When her true type is private information, a rational type may want to develop a reputation as a commitment type by mimicking the commitment type's actions. But, the uninformed player, anticipating the behavior of the rational type, may try to "screen out" the rational type by choosing an action which gives the rational type a low payoff when she mimics the commitment type. My main result shows that for "comparably" patient players, if the prior probability that the player is a commitment type is sufficiently small, the "screening" process may take so long that the rational type does not benefit from developing a reputation. In the case of equally patient players, I show that the folk theorem holds even when both players possess a small amount of private information. Schmidt (1994) and Cripps, Schmidt and Thomas (1993) argue that reputation effects can rule out outcomes permitted by the folk theorem, regardless of how small the prior probability that the player is a commitment type. My results show that this argument only applies when one player is "infinitely" more patient than the other.

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1 Introduction

In this essay, I study the effects of reputation in a class of incomplete-information, two-person, infinitely-repeated games. They differ from standard perfect-information repeated games in that one of the players possesses private information as to whether she is a rational or "commitment" type who plays a fixed repeated-game strategy. Because he does not know the true type of the informed player, the uninformed player can only form a belief about the informed player's type based on her past actions. As a result, a rational informed player can try to develop a "reputation" as a commitment type by mimicking her behavior. The goal is to study whether the possession of a small amount of private information allows the informed player to obtain a higher payoff than in the case of complete information.

Most previous work in the literature consider the case where the uninformed player's discount factor is fixed as the informed player's approaches one. As the informed player's discount factor increases, she would become relatively, and in the limit *infinitely,* more patient than the uninformed player. Fudenberg and Levine (1989) show that in a game in which a long-run player plays against a sequence of short-run players, if the long-run player is patient enough, she will obtain a payoff close to or better than her Stackelberg payoff, which generally is strictly higher than her minmax payoff. Cripps, Schmidt and Thomas (1993) extend the results of Fudenberg and Levine (1989) to the case of two long-run players. They show that, in general, as the informed player becomes sufficiently more patient than her opponent, she can guarantee herself a payoff strictly higher than her minmax payoff. Under some conditions, she can guarantee herself a payoff close to her Stackelberg payoff (Schmidt 1993), and in some others, close to her highest repeated-game payoff (Celentani, Fudenberg, Levine, and Pesendorfer 1993).

The condition of infinite relative patience is obviously highly restrictive. Many economic relationships, for example, repeated oligopolistic competition, involve parties that are equally or, at least, comparably patient. In this paper, I study reputation effects when the informed player is only finitely more patient than the uninformed player in the limit, or, put differently, where the players are equally or *comparably* patient. My main result shows that in any two-person, infinitely-repeated game, except for two special classes specified below, if the two players are equally and sufficiently patient, the commitment strategy is history-independent, and the prior probability of that the informed player is a commitment type is sufficiently small, then any strictly individually rational payoff profile can be supported by a perfect Bayesian equilibrium. Under slightly stronger conditions, the same result applies even when the commitment strategy is history-dependent, the players are comparably (but not equally) patient, and both players possess private information. In other words, the minimum equilibrium payoffs that the informed player may receive with or without private information are the same. In conclusion, reputation effects do not exist when the players are comparably patient, and the prior probability that the informed player is a commitment type is sufficiently small. Compared to previous results, my results suggest that the strength of reputation effects critically depends on the players' relative patience. For any fixed prior probability that a player is a commitment type, reputation effects become stronger as the informed player becomes more patient than her opponent. But when the relative patience between players is fixed, reputation effects diminish as the prior probability that the player is a commitment type decreases.

Call the informed player Player 1 and the uninformed player Player 2. In equilibrium, Player 2 should expect that a rational Player 1 may want mimic the commitment type. As a result, Player 2 may choose an action which will give the rational Player 1 mimicking the commitment type a low payoff so as to "screen out" the rational type. "Screening" is rational for Player 2 if he believes that the rational type of Player 1 may reveal her type in the future and that in this case he will obtain a higher continuation payoff. Suppose T is the maximum number of screening periods that is consistent with some equilibrium.[1] If Player 1 plays the commitment strategy indefinitely, she will receive a low average payoff v_1^s during the screening periods and a high average payoff v_1^* after the screening periods, when Player 2 is convinced that she is a commitment type, and her average discounted payoff will be equal to

$$(1-\delta_1^T)v_1^s + \delta_1^T v_1^*,$$

where δ_1 is Player 1's discount factor. If δ_1^T is small, her average payoff is approximately equal to the low payoff during the screening periods, and there is little to gain from developing a reputation. When Player 2 is more patient, he is willing to screen for more periods; therefore, T is an increasing function of δ_2, Player 2's discount factor. Notice that the cost of screening to Player 2 is at most equal to $(1-\delta_2^T)d$ where d is equal to the largest difference in payoffs between any two stage-game outcomes. Player 2 is willing to pay some positive cost to screen whenever he expects to receive some positive long-run benefit in the event that the screening is successful; therefore, δ_2^T should be less than 1 in the limit. Furthermore, the limit of δ_2^T will be small when it is unlikely that Player 1 is a commitment type. The relative patience of the players matters because it determines the relative sizes of δ_1^T and δ_2^T. When Player 1 and Player 2 are comparably patient in the limit, if

[1] T is finite because Player 2 will screen in a period only if he expects the rational type of Player 1 will reveal her type with a probability bounded away from zero. See Section 2 for details.

the limit of δ_2^T is small, then the limit of δ_1^T will be small as well. This explains why the results of Cripps, Schmidt and Thomas (1993) and others do not apply to this case.

There are two classes of games which are exceptions to the above argument. In both cases, there must exist a stage-game action a_1 for Player 1 such that if Player 1 chooses a_1 and Player 2 chooses a best response to it, then Player 1 receives her highest payoff in the convex hull of the set of strictly individually rational stage-game payoffs. In this case, Player 1 will obtain her highest repeated-game payoff if she can credibly commit to choosing a_1. A stage game is called a *strongly-conflicting-interest game* if Player 2 will get his minmax payoff when he best responds to a_1. A stage game is called a *strongly-dominant-action game*, and a_1 is called a *strongly-dominant action*, if a_1 is a dominant action for Player 1 and the unique best response against any Player 2's action that gives a Player 1 choosing the commitment strategy a payoff less the commitment payoff.[2] Notice that the argument above critically depended on that the rational type of Player 1 can be induced to reveal her type and that Player 2 will suffer a long-term loss for not screening. A strongly-dominant-action game violates the first condition, while a strongly-conflicting-interest game violates the second.[3]

In section 5, I show that in an infinitely-repeated strongly-dominant-action game with one-sided incomplete information, if the only commitment type of the informed player (Player 1) is one who always chooses the strongly-dominant action, then Player 1 will receive her commitment payoff in any perfect Bayesian equilibrium. Notice that in the perfect-information version of the game, there usually exists a large set of equilibrium outcomes. In this case, reputation effects select a unique outcome (the one most favorable to Player 1) from that set. Moreover, unlike all previous results on reputation effects in infinitely-repeated games, which critically depend on the requirement of infinite relative patience in the limit, this result holds for any strictly positive prior probability of a commitment type, and for any discount factors less than 1. In fact, it applies even when Player 1 is less patient than Player 2. The basic argument is as follows: Suppose in some equilibrium Player 2 chooses to screen in some period; then he must believe that the rational type of Player 1 will reveal her type with a finite probability in the future. But in strongly-dominant-action games, the rational type of Player 1 will reveal her type only when she expects Player 2 to screen in the future. Hence, by repeating the same argument, we can conclude that in such an equilibrium Player 2 has to choose to screen in an infinite number of periods. The key of the proof is to show that this cannot happen because eventually Player 2 will be convinced that Player 1 is a commitment

[2] These terms are formally defined in section 4.

[3] I would like to thank Eddie Dekel and Wolfgang Pesendorfer for pointing out the second point to me.

type. Hence, any screening by Player 2 is not consistent with a perfect Bayesian equilibrium and, thus, the rational type of Player 1 must receive her commitment payoff in any perfect Bayesian equilibrium. Notice that this argument is different from the one introduced by Fudenberg and Levine (1989) and commonly used in the reputation literature. That argument only requires that Player 2 update his beliefs rationally and behave optimally given his beliefs, while my argument, in addition, also makes use of the fact that the strategy of the rational type of Player 1 is a best response to Player 2's strategy in equilibrium.

There are two recent papers which address issues similar to those in this one.[4] Cripps and Thomas (1997) demonstrate that reputation effects do not exist in a class of two-person infinitely-repeated common-interest games with equally patient players. They show that when the players are sufficiently patient, and one player may be a commitment type who always plays the Pareto-dominant action, there exists a perfect Bayesian equilibrium in which the payoff for the informed player is close to her minmax payoff. The general idea of their proof is similar to that of Theorem 1, but their result only applies when the stage game belongs to a special class of common-interest games. Cripps (1997) studies infinitely-repeated games with one-sided incomplete information. He shows that when uncertainty is small and the players are sufficiently patient, any strictly individually-rational payoffs can be supported by a perfect Bayesian equilibrium. Despite apparent similarities, his model is not about reputation effects. Unlike other recent work on this topic, including this one, which assumes that the commitment type plays a fixed commitment strategy, Cripps assumes that the commitment type possesses a stage-game payoff function different from that of the rational type.[5] Since he specifically rules out by assumption the possibility that a commitment type will choose a fixed repeated-game strategy, his results do not apply to any of the cases studied in this paper.[6]

The rest of the paper is organized as follows: In Section 2, I introduce a measure of relative patience, and apply it to study the relationship between relative patience and reputation effects.

[4] My results are independently of these.

[5] In Cripps' model, all types are rational. I continue to use the term "commitment type" and "rational type" for convenience only.

[6] For example, consider a simple commitment type who chooses a fixed stage-game action independent of the strategy of her opponent. If we want to model a simple commitment type as a rational player who has a different stage-game payoff function, the payoff function must give the commitment type her highest stage-game payoff whenever she chooses the commitment action (regardless of what the other player chooses). If not, when the commitment player is patient enough, there are always some repeated-game strategies for the other player which will induce the commitment player to deviate from the commitment action. This implies that the minmax payoff for the commitment type is equal to her highest stage-game payoff, and the set of her individually-rational payoffs is a singleton. Assumption A.1 in Cripps (97) rules out any payoff functions with this property. This explains why the two exceptions discussed in this paper do not appear in his model.

This section makes clear that the qualitative results which hold in the simple case of equally patient players also apply to the more general case of comparably patient players. In Sections 3, 4, and 6, I assume the players are equally patient. But under a minor technical assumption, the results in these sections apply to the case of comparably patient players as well. Section 3 introduces a model of two-person infinitely-repeated games with one-sided incomplete information. In Section 4, I establish the main result of this paper: Under fairly general conditions, when the prior probability that the informed player is a commitment type is sufficiently small, there exists a perfect Bayesian equilibrium in which the payoff for the informed player is close to her minmax payoff. Section 5 considers an important exception to the general result that reputation effects do not exist in two-person infinitely-repeated games. I show that in the case of infinitely-repeated strongly-dominant-action games, if there is a slight probability that a player is a commitment type who always chooses the strictly-dominant action, then that player will receive her commitment payoff in any perfect Bayesian equilibrium. In Section 6, I extend the result in Section 4 and prove a folk theorem with two-sided incomplete information. Section 7 contains the conclusion.

2 Relative Patience and Reputation Effects

Absolute patience and relative patience are two distinct concepts. One player can be a lot more patient than another, even when both are very patient. Consider any strictly increasing, continuously differentiable function $\delta_2 : [0,1] \to [0,1]$ such that $\lim_{\delta_1 \to 1} \delta_2(\delta_1) = 1$. This function $\delta_2(\delta_1)$ expresses Player 2's discount factor as a function of Player 1's. The graph $\{\delta_1, \delta_2(\delta_1)\}_{\delta_1=0}^{1}$ defines a path ending at $(1,1)$ on the unit square. The players become more patient as the discount factors move along a path toward $(1,1)$. Conventional folk theorems, assuming that the players are equally patient, characterize the limiting set of equilibrium payoffs as the common discount factor approaches one along the diagonal. But in general, the discount factors of the players need not be the same along a path.

I measure the relative patience between the players by $m = \frac{\ln \delta_1}{\ln \delta_2}$, the ratio of the logarithm of their discount factors. The ratio m compares δ_1^t and δ_2^t, the weights the players put on their continuation payoffs after period t. For any t, $\delta_1^t = (\delta_2^t)^m$. When m is less than one, δ_1^t is larger than δ_2^t and Player 1 is therefore more patient than Player 2.

Definition 1 *Along any path $\{\delta_1, \delta_2(\delta_1)\}_{\delta_1=0}^{1}$, Player 1 is infinitely more patient than Player 2 in* *the limit if*

$$\lim_{\delta_1 \to 1} m(\delta_1, \delta_2) = 0$$

The relative patience between the players may change as the discount factors move along a path defined by $\delta_2(\delta_1)$. Player 1 is infinitely more patient than Player 2 if $m(\delta_1, \delta_2)$ approaches 0 in the limit. Geometrically, the definition means that the slope of $\delta_2(\delta_1)$ approaches infinity as δ_1 goes to one. I call the players *comparably patient* in the limit if neither player is infinitely more patient than the other.

Whether a number of periods are "long" for a player depends on her discount factor. One hundred periods may be "short" for a player with a discount factor of 0.9999 but very "long" for a player with a discount factor of 0.9. This notion of "length" is captured by δ_i^t. T periods are "longer" for Player 1 than for Player 2 if δ_1^T is less than δ_2^T. Lemma 2.1 says that along any path, a number of periods that are arbitrarily "long" for Player 2 becomes arbitrarily "short" for Player 1 in the limit if and only if Player 1 is infinitely more patient than Player 2.[7] The following example illustrates the relation between relative patience and reputation effects.

Lemma 1 *Player 1 is infinitely more patient than Player 2 in the limit as δ_1 goes to 1 iff*

$$\forall\ \epsilon, \eta \in (0,1),\ \exists\ \underline{\delta}_1\ s.t.\ \forall\ \delta_1 \geq \underline{\delta}_1,\ and\ \ \forall\ t \in \Re,\ \delta_2(\delta_1)^t = \epsilon \Rightarrow \delta_1^t \geq\ 1 - \eta.$$

		Player 2	
		D	C
Player 1	D	-d,-d	q,0
	C	0,q	q,q

Figure 1: Example 1

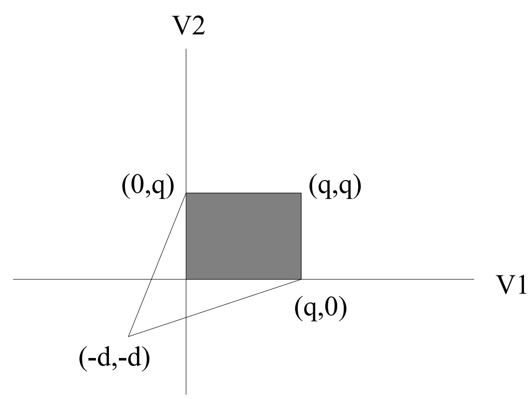

Figure 2: Stage-game payoffs

In the stage game depicted in Example 1, there are three pure strategy equilibria: (C,D), (C,C) and (D,C). Let α denote the probability that Player 1 chooses D and β the probability that Player 2 chooses D. The set of mixed strategy equilibria is equal to $\{(\alpha, \beta) : \alpha = 1 \text{ or } \beta = 1\}$. The convex hull of the set of feasible stage-game payoffs is shown in Fig. 2. The set of equilibrium

[7]The proofs of all lemmas in this section are given in the appendix.

payoffs coincides with the Pareto frontier of the feasible payoff set.[8] If the game is played once, in equilibrium at least one player receives a payoff of q. But when the game is repeated indefinitely and the players are sufficiently, as the minmax payoffs for both players are 0, any strictly positive payoff profile can be supported by subgame perfect Nash equilibrium. The shaded area in Figure 2 denotes the set of equilibrium payoffs of the infinitely-repeated game.

Suppose there is some small probability μ_0 that Player 1 is a commitment type always choosing D. Player 2's best response against such a commitment type is to play C. As Player 1's type is private information, she can try to develop a reputation as a commitment type by choosing D. If a rational Player 1 convinces Player 2 that she is a commitment type, she will receive a payoff of q. But in equilibrium Player 2 can try to screen out the rational type of Player 1 by choosing D. If he and the rational Player 1 both choose D, then each will receive $-d$. Choosing D is costly to Player 2 as well for he can obtain 0 instead of $-d$ by choosing C. Thus, Player 2 will choose to screen only if he expects a rational Player 1 to reveal her type either in that or some future period. And if Player 1 continues to choose D, Player 2 eventually will be convinced that Player 1 is a commitment type. Screening thus cannot last indefinitely. Whether Player 1 can gain from developing a reputation depends on how long the screening phase can last. If the screening phase is short, Player 1 can obtain a payoff close to the commitment payoff by immitating the commitment type. But if the screening phase is long, the cost for developing a reputation may be so high that it is not worthwhile to do so.

To understand the connection between the length of the screening phase and the players' discount factors, imagine the rational type Player 1 chooses the following strategy: In each period t if she has not chosen C before, she will choose C with probability p_t and D with probability $1-p_t$. If she chooses C, she reveals that she is rational and her continuation strategy depends on Player 2's action in that period. If Player 2 chooses D, she will choose C in all future periods, giving Player 2 a continuation payoff of q. But if Player 2 chooses C, she will always choose D, and Player 2's continuation payoff will be 0.

Given Player 1's strategy, Player 2 pays a single period cost slightly less than d for choosing C, and in return he receives a long-term gain of q if Player 1 chooses C. Let μ_t be Player 2's posterior belief that Player 1 is a commitment type in period t. Player 2 prefers to choose D only if expected

[8] This example is non-generic, but the conclusion I draw applies generally. A more general treatment is given in the Sections 3 and 4.

long-term gain outweigh the short-run loss:

$$(1-\delta_2)d \leq \delta_2 p_t(1-\mu_t)q. \tag{1}$$

To satisfy Equation 1, p_t cannot be too small. To induce Player 2 to screen, the rational type of Player 1 must reveal her type with a probability bounded away from zero. Set $p_t = \frac{\Delta}{1-\mu_t}$, with

$$\Delta = \frac{(1-\delta_2)d}{\delta_2 q}. \tag{2}$$

As the rational type of Player 1 chooses D with probability less than one, Player 2 assigns a higher posterior belief that Player 1 is a commitment type every time Player 1 chooses D. Specifically, his posterior belief is given by:

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1-\mu_t)(1-p_t)} = \frac{\mu_1}{(1-\Delta)^t}. \tag{3}$$

Equation 3 shows that μ_t is an increasing in t . As μ_{t+1} cannot exceed one, Equation 6 imposes an upper bound on the number of screening periods. For any $\bar{\mu} < 1$, define

$$T = \max\{t : \frac{\mu_1}{(1-\Delta)^t} \leq \bar{\mu}\}.[9] \tag{4}$$

Given Player 1's strategy, it is consistent with Bayesian learning for Player 2 to choose D for T periods. In this case, if a rational Player 1 chooses the commitment strategy indefinitely, she will receive $-d$ in each of the first T periods and q in all subsequent ones. Her average payoff, v_1^*, is equal to

$$v_1^* = (1-\delta_1^T)(-d) + \delta_1^T q.$$

Whether Player 1 benefits from developing a reputation depends on the value of δ_1^T. Player 1 receives a payoff close to the commitment payoff by mimicking the commitment type when δ_1^T is close to one, but not when δ_1^T is close to zero. When the players become more patient, Player 1 has a stronger incentive to develop a reputation, but Player 2 also has a stronger incentive to screen. Formally, along any path $\{\delta_1, \delta_2(\delta_1)\}_{\delta_1=0}^1$, we can define $\Delta(\delta_1)$, $\mu_t(\delta_1)$, and $T(\delta_1)$ according to Equations 2, 3 and 4. $\Delta(\delta_1)$ is decreasing in δ_1 and goes to zero as δ_1 goes to one. When Player 2 is more patient, he puts more weight on his long-run payoff and, therefore, can be induced to screen even when there is only a small probability that Player 1 will deviate from the commitment strategy. As a result, $T(\delta_1)$ is increasing in δ_1. Lemma 2.2 shows that in the limit these two opposing effects balance out, and δ_1^T depends on the relative patience of the players.

Lemma 2 *Along any path* $\{\delta_1, \delta_2(\delta_1)\}_{\delta_1=0}^1$,

1. $\lim_{\delta_1 \to 1} \delta_2(\delta_1)^{T(\delta_1)} = \frac{\mu_0}{\bar{\mu}}^{\frac{q}{d}}$.

2. *If* $\lim_{\delta_1 \to 1} m(\delta_1, \delta_2(\delta_1)) = \bar{m} > 0$, *then* $\lim_{\delta_1 \to 1} \delta_1^{T(\delta_1)} = \lim_{\delta_1 \to 1} (\delta_2(\delta_1)^{T(\delta_1)})^{\bar{m}} = \frac{\mu_0}{\bar{\mu}}^{\frac{q\bar{m}}{d}}$.

Lemma 2.2 illustrates how various factors affect the length of the screening phase relative to the patience of the players. The variables q, d, and μ_0 jointly determine $\lim_{\delta_1 \to 1} \delta_2(\delta_1)^{T(\delta_1)}$, which, together with $\bar{m}$, in turn determines $\lim_{\delta_1 \to 1} \delta_1^{T(\delta_1)}$. Note that $\lim_{\delta_1 \to 1} \delta_2(\delta_1)^{T(\delta_1)}$ is smaller the smaller the prior belief that Player 1 is a commitment type, the smaller the short-term loss in screening, and the larger the long-term gains. When Player 1 is relatively more patient than Player 2, reputation effects are magnified as $\lim_{\delta_1 \to 1} \delta_1^{T(\delta_1)}$ becomes relatively bigger than $\lim_{\delta_1 \to 1} \delta_2(\delta_1)^{T(\delta_1)}$. When Player 1 is infinitely more patient than Player 2 in the limit, for any fixed $\mu_0 > 0$, $\lim_{\delta_1 \to 1} \delta_1^{T(\delta_1)} = 1$. As a result, Player 1 will always receive a payoff arbitrarily close to the commitment payoff, no matter how small the prior probability that Player 1 is a commitment type. However, the same is not true when the players are comparably patient in the limit. The next lemma shows that in that case, for any q, d, and $\bar{\mu}$, $\lim_{\delta_1 \to 1} \delta_1^{T(\delta_1)}$ is close to 0 when μ_0 is sufficiently small.

Lemma 3 *If* $\lim_{\delta_1 \to 1} m(\delta_1, \delta_2(\delta_1)) = \bar{m} > 0$, *then* $\forall\ \epsilon > 0\ \exists\ \bar{\mu_0}$ *and* $\exists\ \underline{\delta_1}$, *such that* $\forall\ \mu_0 \leq \bar{\mu_0}$ *and* $\forall\ \delta_1 \geq \underline{\delta_1}$, $\delta_1^{T(\delta_1)} \leq \epsilon$.

As Lemma 2.3 applies whenever q and $\bar{m}$ is strictly positive, whether the players are equally patient or comparably (but not equally) patient does not make any significant difference.[10] In either case, if the prior probability that Player 1 is a commitment type is sufficiently small, it is possible for a rational Player 1 to reveal her type with a probability that is large enough to induce Player 2 to choose a non-best-response to the commitment strategy, but yet small enough so that screening can potentially last for a long time. In Sections 3, 4 and 6, I assume that the players are equally patient, but, barring a minor technical assumption specified in Section 4.3, all results apply to the case of comparably patient players as well.

3 A Model

3.1 Preliminaries

Consider a two-person, infinitely-repeated game $\Gamma(A, g, \delta)$, where (A, g) is the stage game and δ is the players' common discount factor. In each period the two players, Players 1 and 2, play a

[10] Of course, for a given ϵ, the threshold $\bar{\mu}_0$ is smaller the smaller q and $\bar{m}$.

simultaneous-move game (A, g). $A = A_1 \times A_2$ denotes the finite set of stage-game actions and $g = (g_1, g_2)$ denotes the payoff functions. A pure action, a mixed action, and the set of mixed actions of Player i are denoted as a_i, α_i, and $\mathcal{A}_i$, respectively. The expected payoffs for a mixed action profile α is denoted by $g(\alpha) = (g_1(\alpha), g_2(\alpha))$. Define $br_j(a_i)$ as the set of Player j's best responses against a_i. The set V is the convex hull of the feasible stage-game-payoff set. A payoff profile $v = (v_1, v_2)$ is strictly individually rational if each of v_i, $i \in \{1, 2\}$, is strictly larger than $\underline{v}_i$, the minmax payoff for Player i. The set $V^* \equiv \{(v_1, v_2) \in V : v_i > \underline{v}_i, i = 1, 2\}$ consists of those members of V that are strictly individually rational, and the set V_i^* is the set of strictly individually rational payoffs for Player i.[11] Define $\bar{v}_i = \max V_1^*$ as the highest payoff for Player 1 consistent with Player 2 getting a payoff not lower than his minmax payoff. I assume that V^* is non-empty and the closure of V_i^* contains $\underline{v}_1$ and $\underline{v}_2$.[12]

In the beginning of each period, the players observe their opponent's last period action. A history of the game, $h = (h^1, h^2)$, is a function from the set of positive integers to $A_1 \times A_2$ so that $h^i(s)$ is Player i's period s action. H_∞ is the set of all histories. I use $h_t = (h_t^1, h_t^2)$ to denote a t-period history that describes the actions of the players up to, but not including, period t. The concatenation of two histories h_t and h_s is represented by $h_t.h_s$.[13] The set of feasible t-period histories is denoted by H_t, and the set of all finite histories is denoted by H.

A pure strategy of Player i in the repeated game is a function $s_i : H \rightarrow A_i$ so that $s_i(h_t)$ is Player i's period-t action after history h_t, and a mixed strategy of Player i is a function $\sigma_i : H \rightarrow \mathcal{A}_i$ so that $\sigma_i(h_t)$ is Player i's period t mixed action after history h_t. Let S_i and Σ_i be the sets of pure and mixed repeated-game strategies of Player i, respectively. The continuation strategy of a strategy x after history h_t is represented by $x|h_t$. A pure-strategy profile (s_1, s_2) induces a history, denoted by $h(s_1, s_2)$. A mixed-strategy profile (σ_1, σ_2) induces a probability distribution P_{σ_1, σ_2} over H_∞. Define $H(\sigma_1, \sigma_2)$ and $H(\sigma_i)$ as the set of histories reachable under (σ_1, σ_2) and under σ_i, respectively.

The players' objectives are to maximize their expected payoffs. When the players choose a strategy profile σ, their expected payoffs, $v(\sigma) = (v_1(\sigma), v_2(\sigma))$ is equal to $(E_\sigma[v_1(h)], E_\sigma[v_2(h)])$ where $v_i(h) = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} g_i(h(t))$. In any period t, the continuation strategy in the next period, $\sigma|h_{t+1}$, is a function of current actions. Let $E_{\sigma(h_t)}[v(\sigma|h_{t+1})]$ represent the players' expected

[11] $V_1^* \equiv \{v_1 : \exists v_2 \text{ s.t. } (v_1, v_2) \in V^*\}$, and $V_2^* \equiv \{v_2 : \exists v_1 \text{ s.t. } (v_1, v_2) \in V^*\}$.

[12] These assumptions simplify the exposition and are not crucial to my results.

[13] Formally, $h_t.h_s$ is a $t + s - 2$ period history such that $h_t.h_s(r)$ is equal to $h_t(r)$ if $r \leq t - 1$ and is equal to $h_s(r - t + 1)$ if $t \leq r \leq t + s - 2$.

continuation payoffs for the players conditioned on h_t and the behavioral strategy in period t.

In complete-information repeated games, the Perfect Folk Theorem (Fudenberg and Maskin 1986, 1990, 1991) implies that any strictly individually rational payoff profile $v \in V^*$ can be supported subgame-perfect Nash equilibrium when the players' discount factors are equal and close to one. There are many slightly different versions of the theorem. To avoid confusion, I include the version I use.

Perfect Folk Theorem (Fudenberg and Maskin)

Consider a two-person infinitely-repeated game in which public randomization is not available and only the players' choices of action are observable. For any $v'_1 \in V_1^*$ and $v'_2 \in V_2^*$, there exists a $\underline{\delta} < 1$ such that for all $\delta \in [\underline{\delta}, 1)$ and for all $v \in \{v : V^* \text{ s.t. } v \geq (v'_1, v'_2)\}$, there is a subgame-perfect Nash equilibrium of the infinitely-repeated game with discount factor δ in which the discounted payoffs are v.

This is essentially Proposition 2 in Fundenberg and Maskin (1991). Here I emphasize that for any strictly-individually-rational payoff profile v, there is a common lower bound $\underline{\delta}$ such that, for any δ higher than $\underline{\delta}$, any payoff profiles weakly dominating v can be supported by subgame-perfect Nash equilibrium, as the "punishment" equilibria that support v would also support any equilibrium payoffs weakly higher than v.

3.2 Repeated Games with Commitment Types

Suppose there is a probability $1 - \mu_0$ that Player 1 is rational and a probability μ_0 that Player 1 is a commitment type γ_1. A commitment type chooses a fixed strategy. Henceforth, γ_1 refers to either the commitment type or the commitment strategy chosen by her. Player 1's type is her private information. Player 2 forms a belief $\mu(h_t)$ about Player 1's type based on history h_t. By definition, $\mu(h_1) = \mu_0$. Given his belief, Player 2 expects Player 1 to choose $\rho_1 \equiv (1-\mu_0)\sigma_1 + \mu_0\gamma_1$. I denote the one-sided incomplete-information repeated game by $\Gamma(A, g, \delta, \gamma_1, \mu_0)$.

I allow the commitment strategy to be any pure history-dependent repeated-game strategy. For any commitment strategy γ_1, the commitment payoff is the minimum payoff a rational Player 1 may receive if she plays γ_1 and Player 2 plays a best response to γ_1. Let $b_2(\gamma_1)$ denote the best response of Player 2 that gives Player 1 the commitment payoff. Formally, the commitment payoff,

$\nu_\delta(\gamma_1)$, is defined as

$$\nu_\delta(\gamma_1) \equiv \min_{b_2 \in B_{2,\delta}(\gamma_1)} v_1(\gamma_1, b_2)$$

where $B_{2,\delta}(\gamma_1) \equiv \arg\max_{s_2 \in S_2} v_2(\gamma_1, s_2)$ is Player 2's best-response set against γ_1. Player 1 will receive a payoff not lower than the commitment payoff, if Player 2 is convinced that Player 1 is a commitment type. In general, the commitment payoff may depend on δ as $B_{2,\delta}(\gamma_1)$ is a function of δ. When γ_1 is history dependent, the commitment payoff after history h_t, denoted by $\nu_\delta(\gamma_1|h_t)$, may also be history dependent. Obviously, Player 1 will never imitate the commitment type if the commitment payoff is lower than the minmax payoff. Without loss of generality, I shall assume that the commitment payoff after any history h_t is larger than $\underline{v}_1$. For simple commitment types that always chooses a fixed stage-game action, the commitment payoff is independent of δ, and I simply denote it by $\nu(\gamma_1)$.[14]

Definition 2 (Perfect Bayesian Equilibrium) $(\sigma_1, \sigma_2, \mu)$ *is a perfect Bayesian equilibrium of an infinitely-repeated game* $\Gamma(A, g, \delta, \gamma_1, \mu_0)$ *if the following conditions hold for all* $h_t \in H$*:*

$$\mu(h_t) = \frac{\mu(h_{t-1})\gamma_1(a_1^{t-1}|h_{t-1})}{\rho_1(a_1^{t-1}|h_{t-1})}$$

whenever the denominator is non-zero,

$$v_1(\sigma_1|h_t, \sigma_2|h_t) \geq v_1(\sigma_1', \sigma_2|h_t) \quad \forall \sigma_1' \in \Sigma_1,$$

$$v_2(\rho_1|h_t, \sigma_2|h_t) \geq v_2(\rho_1|h_t, \sigma_2') \quad \forall \sigma_2' \in \Sigma_2.$$

Perfect Bayesian equilibrium put no restriction on beliefs in information sets off the equilibrium path. In this paper, I impose an extra restriction that $\mu_t(h_t) = 0$ in those information sets, so that whenever Player 2 observes an action of Player 1 that is consistent neither with the rational nor the commitment type, he believes with certainty that Player 1 is a rational. The restriction implies that once a rational Player 1 chooses a non-commitment action, the continuation game becomes one of complete information. It allows me to construct perfect Bayesian equilibrium in a simple way. Instead of specifying the players' strategies and beliefs in all information sets, I need only to do so in information sets in which Player 2 is uncertain about Player 1's type, and specify the continuation payoffs in information sets in which Player 2 learns that Player 1 is rational.

$$\begin{aligned} \text{Define } H(\gamma_1) &= \{h_t : \exists s_2 \in S_2 \text{ s.t. } h_t = h_t(\gamma_1, s_2)\}, \\ \text{and } G(\gamma_1) &= \{h_{t_1}.a : h_{t_1} \in H(\gamma_1) \text{ and } h_{t_1}.a \notin H(\gamma_1)\}. \end{aligned}$$

[14] $\nu_{1,\delta}(a_1) \equiv \min_{a_2 \in br(a_1)} g_1(a_1, a_2)$ where a_1 is the commitment action.

$H(\gamma_1)$ is the set of histories consistent with γ_1, and $G(\gamma_1)$ is the set of histories consistent with γ_1 in all periods with the exception of the last. Let $\sigma^* = (\sigma_1^*, \sigma_2^*)$ and μ^* be the "restricted" behavioral strategy profile and beliefs defined in $H(\gamma_1)$, and $\beta^* = (\beta_1^*, \beta_2^*)$ be the continuation payoffs defined in $G(\gamma_1)$. Since Player 2 does not know Player 1's type, he expects Player 1 to choose $\rho_1^* \equiv (1-\mu^*)\sigma_1^* + \mu^*\gamma_1$. Given σ^*, μ^*, and β^*, $v_1^*(h_t)$ and $v_2^*(h_t)$, the continuation payoffs for Players 1 and 2 after history h_t are given by

$$
\begin{aligned}
v_1^*(h_t) &= (1-\delta)\sum_{s=t}^{\infty}\{\sum_{h_s\in H(\gamma_1)} \delta^{s-t} P_{\sigma^*}(h_s|h_t) g_1(\sigma^*(h_s)) + \sum_{h_s\in G(\gamma_1)} \delta^{s-t} P_{\sigma^*}(h_s|h_t)\beta_1^*(h_s)\} \\
v_2^*(h_t) &= (1-\delta)\sum_{s=t}^{\infty}\{\sum_{h_s\in H(\gamma_1)} \delta^{s-t} P_{(\rho_1^*,\sigma_2^*)}(h_s|h_t) g_2(\sigma^*(h_s)) + \sum_{h_s\in G(\gamma_1)} \delta^{s-t} P_{(\rho_1^*,\sigma_2^*)}(h_s|h_t)\beta_2^*(h_s)\}
\end{aligned}
$$

where $P_{(\rho_1^*,\sigma_2^*)}(h_s|h_t)$ is the probability of reaching h_s, $s > t$, conditional on h_t, and $P_{\sigma^*}(h_s|h_t)$ is the probability of reaching h_s conditional on h_t and Player 1 being rational.

Lemma 4 *If the stage game is neither a strongly-conflicting-interest game nor a strongly-dominant-action game, then $\exists\ \hat{a}_2 \in A_2$, such that $g_1(a_1^c, \hat{a}_2 < \bar{v}_1$ and for some $\hat{a}_1 \in A_1/a_1^c$, $g_1(\hat{a}_1, \hat{a}_2) \geq g_1(a_1^c, \hat{a}_2)$.*

Proof of Lemma 3.1

Given that σ^*, β^*, and μ^*, a perfect Bayesian equilibrium (σ, μ) with the same equilibrium payoffs can be constructed by setting $\sigma(h_t) = \sigma^*(h_t)$ for all $h_t \in H(\gamma_1)$ and $\sigma|h_t = \tilde{\sigma}_{h_t}$ for all $h_t \in G(\gamma_1)$. □

4 Non-Existence of Reputation Effects

The example in Section 2 demonstrates that if the prior probability that Player 1 (the informed player) is a commitment type is small, it may take a long time for her to develop a reputation as a commitment type. In this section, I formalize that intuition and show that in repeated games in which one player possesses a small amount of private information, under fairly general conditions, for any $v_1 \in V_1^*$ there exists a perfect Bayesian equilibrium in which the expected equilibrium payoff for the rational type of the Player 1 is equal to v_1. In other words, when the players are sufficiently patient, any payoff for Player 1 that can be supported by a subgame-perfect Nash equilibrium in a perfect-information repeated game can also be supported by some perfect Bayesian equilibrium when Player 1 possesses a small amount of private information.[15]

[15] Note that the perfect-Bayesian-equilibrium payoff for Player 1 can be arbitrarily close to $\underline{v}_1$as $\underline{v}_1$ is assumed to be containted in the closure of V_1^*.

This section is divided into three parts: The first part considers the case of simple commitment strategies; the second considers the general case of history-dependent commitment strategies; and the last briefly discusses other extensions of the basic result.

4.1 Simple Commitment Types

Theorem 1 shows that when the commitment type is simple, reputation effects do not exist if the stage game is neither a strongly-conflicting-interest game nor a strongly-dominant-action game.

Definition 3 (Strongly-Conflicting-Interest Games) *A stage game (A, g) is a strongly-conflicting-interest game if*
3.1. There exists $\tilde{a}_1 \in A_1$ such that $\forall a_2 \in br(\tilde{a}_1)$, $g_1(\tilde{a}_1, a_2) = \bar{v}_1$ and $g_2(\tilde{a}_1, a_2) = \underline{v}_2$, and
3.2. $\underline{v}_2 = \max\{v_2 : (\bar{v}_1, v_2) \in V\}$.

By definition, $\bar{v}_1 = \max V_1^*$, is the highest payoff for Player 1 consistent with Player 2 getting a payoff not lower than his minmax payoff. Condition 3.2 implies the converse: In strongly-conflicting-interest games, $\underline{v}_2$ is the highest payoff for Player 2 consistent with Player 1 getting $\bar{v}_1$. Condition 3.1 says that there is an action $\tilde{a}_1$ for Player 1 such that if Player 1 chooses $\tilde{a}_1$ and Player 2 chooses a best response to $\tilde{a}_1$, then Player 1 receives $\bar{v}_1$ and Player 2 receives $\underline{v}_2$. In most two-person games, given the action of the other player, a player generally cannot maximize her own payoff and minimize her opponent's simultaneously. But in strongly-conflicting-interest games, if Player 1 can commit to $\tilde{a}_1$, she can obtain a high payoff for herself and enforce a low payoff on Player 2. The notion of strongly-conflicting-interest games impose stronger restrictions than that of conflicting-interest games introduced by Schimdt (1993). The latter requires only that there be an action for Player 1 such that if Player 1 chooses that action and Player 2 best responds, Player 1 receives her Stackelberg payoff and Player 2 receives his minmax payoff. The Stackelberg payoff is generally less than $\bar{v}_1$. Moreover, conflicting-interest games does not require that $\underline{v}_2$ be the highest payoff consistent with Player 1 getting $\bar{v}_1$.

Definition 4 (Strongly-Dominant-Action Games) *A stage game (A, g) is a strongly-dominant-action game if*
4.1. $\exists\ \tilde{a}_1 \in A_1$ such that $\forall a_2 \in br(\tilde{a}_1)$, $g_1(\tilde{a}_1, a_2) = \bar{v}_1$, and
4.2. $\forall a_2 \in \{a_2 \in A_2 : g(\tilde{a}_1, a_2) < \bar{v}_1\}$, $\forall a_1 \in A_1/\{\tilde{a}_1\}$, $g_1(\tilde{a}_1, a_2) > g_1(a_1, a_2)$.
4.3. $\forall a_2 \in \{a_2 \in A_2 : g(\tilde{a}_1, a_2) \geq \bar{v}_1\}$, $\forall a_1 \in A_1/\{\tilde{a}_1\}$, $g_1(\tilde{a}_1, a_2) \geq g_1(a_1, a_2)$.
The action $\tilde{a}_1$ is called a strongly-dominant action for Player 1

Condition 4.1 is similar to the first part of Condition 3.1 in the definition of strongly-conflicting-interest games, except there is no restriction on Player 2's payoff. Conditions 4.2 and 4.3 require that $\tilde{a}_1$ be a weakly-dominant action and, furthermore, be a strict best response against any action a_2 that gives a Player 1 choosing a_1^c a payoff less than $\bar{v}_1$. In a strongly-dominant-action game, if Player 1 chooses the strictly dominant action and Player 2 chooses a best response, then Player 1 will receive $\bar{v}_1$.

Theorem 5 *In a two-person infinitely-repeated game with one-sided incomplete information and where the players' discount factors are equal, $\Gamma(A, g, \delta, \gamma_1, \mu_0^1)$, if the stage game (A, g) is neither a strongly-conflicting-interest game nor a strongly-dominant-action game, and γ_1 is a simple commitment type, then $\forall v_1 \in V_1^*$, $\exists\ \underline{\delta}$ and $\bar{\mu}_0^1$, such that $\forall \delta \geq \underline{\delta}$ and $\mu_0^1 \leq \bar{\mu}_0^1$, there is a perfect Bayesian equilibrium in which the average discounted payoff for the rational type of Player 1 is equal to v_1.*

It is instructive to understand how Theorem 1 fails when the stage game is either a strongly-conflicting-interest or a strongly-dominant-action game. The key of the proof is to construct an equilibrium in which Player 2 chooses to screen. In such an equilibrium, because of the constraint imposed by Bayesian learning, there is always a final period of screening in which if Player 1 chooses the commitment action in that period, she will receive the commitment payoff from the next period onward. In strongly-dominant-actions games in which the commitment type always chooses the strictly dominant action, a rational Player 1 strictly prefers to choose to imitate the commitment type in the final period of screening as doing so gives her the highest payoff in that period as well as in the continuation game. But if the rational Player 1 never reveals her type in the final screening period, Player 2 would not have any incentive to screen in that period, and, hence, the equilibrium breaks down. In strongly-conflicting-interest games, while it is possible to induce the rational Player 1 to reveal her type, doing so would imply that Player 2 must receive a continuation payoff close to his minmax payoff even if he chooses to screen. As screening is costly and his continuation payoff for not screening could not be made lower than the minmax payoff, Player 2 would not screen.

.

Proof of Theorem 1

As Player 2 can always guarantee himself a payoff not lower than the minmax, the commitment payoff $\nu(\gamma_1)$ can never exceed $\bar{v}_1$. Here, I present the proof for the case where $\nu(\gamma_1) = \bar{v}_1$. As we shall see, it is easier to construct the desired equilibrium for the proof when $\nu(\gamma_1)$ is strictly lower than $\bar{v}_1$, as it is easier to induce a rational Player 1 to reveal her type when the commitment payoff is not the best equilibrium outcome for her. As the case where $\nu(\gamma_1) < \bar{v}_1$ is also covered by Theorem 2, I shall not present a separate proof for it.

I prove the theorem by showing that if μ_0 is sufficiently small, then for every discount factor δ above some threshold, it is possible to construct a restricted strategy profile σ^* and a system of beliefs μ^*, both defined for in $H(\gamma_1)$, and a set of continuation payoffs β^*, defined in $G(\gamma_1)$, such that σ^*, μ^*, and β^* are consistent with perfect Bayesian equilibrium. Implicitly, the σ^*, β^*, and μ^* defined below are all functions of δ.

First, I show that it is possible to construct a perfect Bayesian equilibrium in which the equilibrium payoff for the rational type of Player 1 is equal to $v_1^1 \in [v_1 - (1-\delta)d, v_1] \subset V_1^*$.

Let a_1^c be the commitment action for Player 1, and $a_2^c \in \arg\min_{a_2 \in br(a_1^c)} g_1(\hat{a}_1, a_2)$ be the action in Player 2's best-response set that gives Player 1 her lowest payoff. The first step of the proof is to show that there are actions that Player 2 can use to punish a rational Player 1 for imitating the commitment type. As $\underline{v}_1$ is Player 1's minmax payoff, there is some action a_2^s such that $g_1(a_1^c, a_2^s) \leq \underline{v}_1$. If Player 2 chooses a_2^s, a rational Player 1 will receive a payoff not higher than her minmax payoff in that period by choosing a_1^c. Let $a_1^* \in \arg\max_{a_1 \in A_1/\{a_1^c\}} g_1(a_1, a_2^s)$. Among all non-commitment actions, a_1^* gives Player 1 the highest stage-game payoff against a_2^s. The definition of a_1^* does not mean that a_1^* is a best response to a_2^s, as it is possible that the commitment action a_1^c itself yields a higher payoff than a_1^*. The following lemma says that if the stage game is neither a strongly-conflicting-interest game nor a strongly-dominant-action one, then there is always some action of Player 2 that punishes Player 1 and to which a_1^c is not the unique best response.

Lemma 6 *If the stage game is neither a strongly-conflicting-interest game nor a strongly-dominant-action game, then* $\exists\ \hat{a}_2 \in A_2$ *such that* $g_1(a_1^c, \hat{a}_2) < \bar{v}_1$ *and for some* $\hat{a}_1 \in A_1/a_1^c$, $\hat{a}_1 \in br(\hat{a}_2)$.

The actions a_2^s and $\hat{a}_2$ will be the "punishments" Player 2 uses to screen out a rational Player 1. In equilibrium Player 2 chooses a_2^s during the first T_1 periods and $\hat{a}_2$ during the next T_2 periods, unless Player 1 reveals that she is rational by choosing a non-commitment action. The two phases serve different functions. The first T_1 periods brings the payoff of a rational Player 1 imitating the commitment close to v_1, while the second T_2 periods induce a rational Player 1 to reveal her type when screening ends.[16] If Player 1 chooses the commitment action in each of the first $T_1 + T_2$ periods, Player 2 will choose a_2^c from period $T_1 + T_2$ onward. Player 1's equilibrium payoff v_1^1 is given by

$$v_1^1 = (1-\delta^{T_1})g_1(a_1^c, a_2^s) + \delta^{T_1}\{(1-\delta^{T_2-1})g_1(a_1^c, \hat{a}_2) + \delta^{T_2-1}(1-\delta)g_1(\hat{a}_1, \hat{a}_2) + \delta^{T_2}\bar{v}_1\}$$

[16] If $\nu(\gamma_1)$ is strictly less than $\bar{v}_1$, then it would be unnecessary to have the second T_2 periods.

In equilibrium, a rational Player 1 imitates the commitment type with positive probability. Let Δ be a constant equals to $\frac{(1-\delta)d}{\delta q}$, where d is the largest difference in payoff for either player between any two outcomes. In each of the first $T_1 + T_2$ periods, a rational Player 1 chooses a_1^c with probability $1 - \frac{\Delta}{1-\mu_t^*}$, and reveals her type with a probability of $\frac{\Delta}{1-\mu_t^*}$ by choosing a_1^* in the first T_1 periods and $\hat{a}_1$ in the next T_2. If Player 1 is rational and has chosen a_1^c in each of the first $T_1 + T_2 - 1$ periods, then she will choose $\hat{a}_1$ with certainty in period $T_1 + T_2$.

As both $g_1(a_1^c, \hat{a}_2)$ and v_1 are less than $\bar{v}_1$, there exists δ^* such that for all $\delta \geq \delta^*$, such that $\bar{v}_1 - \frac{1-\delta}{\delta} d \in (\max(v_1, g_1(a_1^c, \hat{a}_2)), \bar{v}_1)$. For all $\delta \geq \delta^*$, I can choose T_1 and T_2 such that:

$$v_1^1 \in [v_1 - (1-\delta)d, v_1],$$
$$(1-\delta^{T_2-1}) g_1(a_1^c, \hat{a}_2) + \delta^{T_2-1}(1-\delta) g_1(\hat{a}_1, \hat{a}_2) + \delta^{T_2} \bar{v}_1 \leq \bar{v}_1 - \frac{1-\delta}{\delta} d.$$

The first condition means that if a rational Player 1 chooses a_1^c in the first $T_1 + T_2 - 1$ periods, $\hat{a}_1$ in period $T_1 + T_2$, and receives a continuation payoff equal to the commitment payoff thereafter, her total payoff will lie between $v_1 - (1-\delta)d$ and v_1. The second condition means that the continuation payoff after T_1 periods for such a Player 1 is not greater than $\bar{v}_1 - \frac{1-\delta}{\delta} d$.

It follows from Lemma 2.3 that there exist $\bar{\mu}_0$ and δ^{**} such that $\forall \mu_0 \leq \bar{\mu}_0$ and $\delta \geq \delta^{**}$, $\frac{\mu_0}{(1-\Delta)^{T_1+T_2}} \leq 1$. For all $\mu_0 \leq \bar{\mu}_0$ and $\delta \geq \max\{\delta_1^*, \delta^{**}\}$, the equilibrium strategy and belief in $H(\gamma_1)$, denoted by (σ_1^*, σ_2^*) and μ^*, are described in Tables 1 and 2.

$\mathbf{h_t \in H(\boldsymbol{\gamma_1})}$	$\boldsymbol{\sigma_1^*}(\mathbf{h_t})$	$\boldsymbol{\sigma_2^*}(\mathbf{h_t})$
$1 \leq t \leq T_1$	$\sigma_1^*(a_1^*\|h_t) = \frac{\Delta}{1-\mu(h_t)}$ $\sigma_1^*(a_1^c\|h_t) = (1 - \frac{\Delta}{1-\mu(h_t)})$	$\sigma_2^*(a_2^s\|h_t) = 1$
$T_1 < t < T_1 + T_2$	$\sigma_1^*(\hat{a}_1\|h_t) = \frac{\Delta}{1-\mu(h_t)}$ $\sigma_1^*(a_1^c\|h_t) = (1 - \frac{\Delta}{1-\mu(h_t)})$	$\sigma_2^*(\hat{a}_2\|h_t) = 1$
$t = T_1 + T_2$	$\sigma_1^*(\hat{a}_1\|h_t) = 1$	$\sigma_2^*(\hat{a}_2\|h_t) = 1$
$t > T_1 + T_2$	$\sigma_1^*(a_1^c\|h_t) = 1$	$\sigma_2^*(a_2^c\|h_t) = 1$

Table 1: The equilibrium strategies, $\sigma^* \quad \forall h_t \in H(\gamma_1)$

$\mathbf{h_t} \in \mathbf{H}$	$\boldsymbol{\mu}^*(\mathbf{h_t})$
$h_t \in H(\gamma_1), \quad 1 \leq t \leq T_1 + T_2$	$\frac{\mu_0^1}{(1-\Delta)^{t-1}}$
$h_t \in H/H(\gamma_1)$	0
$h_t \in H(\gamma_1), \quad t > T_1 + T_2$	1

Table 2: Player 2's beliefs about Player 1's type

Once Player 1 reveals that she is rational, the continuation game becomes one of complete information. Partition $G(\gamma_1)$, the information sets in which Player 2 learns that Player 1 is rational, into two subsets: $G_1 \equiv \{h_t.a^t \in G(\gamma_1) : \ a_2^t \neq \sigma_2^*(h_t)\}$ and $G_2 \equiv \{h_t.a^t \in G(\gamma_1) : \ a_2^t = \sigma_2^*(h_t)\}$. The first subset G_1 contains the information sets in which Player 2 deviates in the preceding period, whereas the second subset G_2 contains information sets in which Player 2 does not deviate in the preceding period.

As the stage game is not a strongly-conflicting-interest game, there exists $v_2' > \underline{v}_2$ such that $(\bar{v}_1, v_2') \in V^*$. By assumption, $\underline{v}_1$ is in the closure of V_1^*, and, thus, there is $v_2'' \geq \underline{v}_2$ such that $(\underline{v}_1, v_2'') \in V$. Let $f : V_1 \to \Re$ be the linear function that represents the line connecting $(\underline{v}_1, v_2'')$ and $(\bar{v}_1, v_2')$. Let $(\check{v}_1, \check{v}_2)$ and $(\hat{v}_1, \hat{v}_2)$ be two points in V^* so that $\check{v}_1 \in (\underline{v}_1, v_1)$ and $\hat{v}_2 \in (\underline{v}_2, \min\{f(v_1), f(\bar{v}_1)\})$. Let $q = \min\{f(v_1), f(\bar{v}_1)\} - f(\hat{v}_1)$. See Figure 3.

Define $\Lambda(t) : \{1, .., T_1 + T_2\} \to \Re$ such that:

$$v_1^1 = (1 - \delta) \sum_{i=1}^{t-1} \delta^{i-1} g_1(a_1^c, \sigma_2^*) + \delta^{t-1} \Lambda(t).$$

$\Lambda(t)$ is the equilibrium continuation payoff for the rational type of Player 1 in period t during the screening phase. The continuation payoffs in $G(\gamma_1)$ are selected to support the strategies described in Table 1. For $h_t \in G_1$, set $\beta^*(h_t) = (\check{v}_1, f(\check{v}_1))$. The continuation payoffs $\beta^*(h_t)$ for $h_t \in G_2$ are given in Table 3.

The "restricted" equilibrium strategies σ_1^* and σ_2^* are sequentially rational given μ^* and β^*. Consider Player 1. In period $T_1 + T_2$, she gets a higher payoff in that period by revealing her type than imitating the commitment type as $g_1(a_1^c, \hat{a}_2) < g_1(\hat{a}_1, \hat{a}_2)$, and regardless of what she chooses, she will get a continuation payoff $\bar{v}_1$ from period $T_1 + T_2 + 1$ onward. It is thus optimal

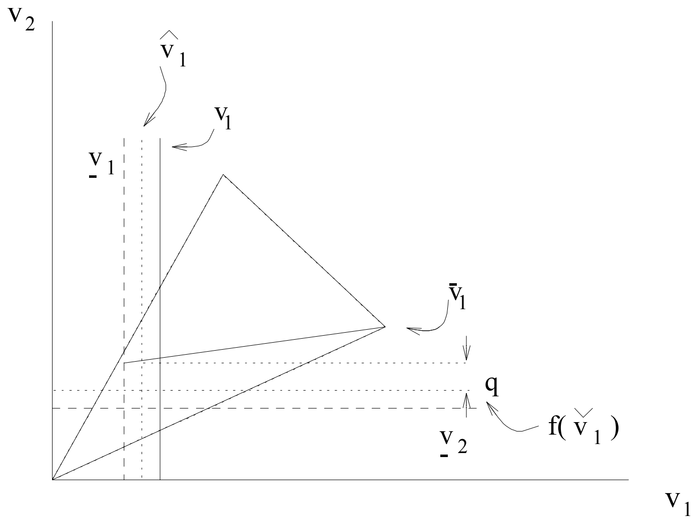

Figure 3: Stage-game Payoff when $\nu(\gamma_1) = \bar{v}_1$

$\mathbf{h_t \in G_2}$	$\boldsymbol{\beta_1^*}(\mathbf{h_t})$	$\boldsymbol{\beta_2^*}(\mathbf{h_t})$
$1 \leq t \leq T_1 + 1$	$\Lambda(t) - \frac{1-\delta}{\delta}(g_1(a_1^c, a_2^s) - g_1(a_1^*, a_2^s))$	$f(\beta_1^*(h_t))$
$T_1 + 2 \leq t \leq T_1 + T_2$	$\Lambda(t) - \frac{1-\delta}{\delta}(g_1(a_1^c, \hat{a}_2) - g_1(\hat{a}_1, \hat{a}_2))$	$f(\beta_1^*(h_t))$
$t = T_1 + T_2$	$\bar{v}_1$	$f(\bar{v}_1)$

Table 3: Continuation Payoffs, $\beta^*(h_t)$, $\quad \forall h_t \in G_2$

for her to choose $\hat{a}_1$ in period $T_1 + T_2$. In any period $t \in [T_1 + 1, T_1 + T_2 - 1]$, if Player 1 chooses a_1^c and follows the equilibrium strategy thereafter, she will receive a payoff of $(1 - \delta)g_1(a_1^c, \hat{a}_2) + \delta\Lambda(t)$; if she chooses $\hat{a}_1$ and follows the equilibrium strategy thereafter, she will receive a payoff of $(1 - \delta)g_1(\hat{a}_1, \hat{a}_2) + \delta\beta_1^*(h_{t-1}.(\hat{a}_1, \hat{a}_2))$. In any period $t \in [1, T_1]$, if Player 1 chooses a_1^c and follows her equilibrium strategy thereafter, she will receive a payoff of $(1 - \delta)g_1(a_1^c, a_2^s) + \delta\Lambda(t)$; if Player 1 chooses a_1^* and follows her equilibrium strategy after that, she will receive a payoff of $(1 - \delta)g_1(a_1^*, a_2^s) + \delta\beta_1^*(h_{t-1}.(a_1^*, a_2^s))$. It is straightforward to verify that $\beta_1^*(h_t)$ is defined so that Player 1 is indifferent between a_1^c and a_1^* in the first T_1 periods, and between a_1^c and $\hat{a}_1$ in the next $T_2 - 1$ periods.

Consider Player 2. By construction, his action in any period t during the screening phase will only affect his stage-game payoff and the continuation payoff in the event that Player 1 reveals that she is rational in the same period. Hence, the short-term cost of screening is less than $(1-\delta)d$, while the long-term gain from screening is larger then $\delta\Delta q$. It is easy to verify that σ_2^* is optimal for Player 2.

Lastly, I need to show that $(\beta_1^*(h_t), f(\beta_1^*(h_t)) \in V^*$ for all $h_t \in G_2$. By construction, if $x \in [\check{v}_1, \bar{v}_1]$, then $(x,\, f(x)) \in V^*$. For all $h_t \in G_2$ $\beta_1^*(h_t) > v_1 > \check{v}_1$. I now show that $\beta_1^*(h_t)$ is less than $\bar{v}_1$. As T_2 is defined to be less than $\bar{v}_1 - \frac{1-\delta}{\delta}d$,

$$\beta_1^*(h_{T_2}) \le \bar{v}_1 - \frac{1-\delta}{\delta}d + \frac{1-\delta}{\delta}(g_1(a_1^c, a_2^s) - g_1(a_1^*, a_2^s)) \le \bar{v}_1.$$

Since $\Lambda(t)$ is increasing in t, if follows that for all $t \le T_1$, $\beta_1^*(h_t) \le \bar{v}_1$. For all $T_1 + 1 \le t \le T_1 + T_2$, since $\Lambda(t) \le \bar{v}_1$ and $g_1(a_1^c, \hat{a}_2) - g_1(\hat{a}_1, \hat{a}_2) < 0$,

$$\beta_1^*(t) = \Lambda(t) - \frac{1-\delta}{\delta}(g_1(a_1^c, \hat{a}_2) - g_1(\hat{a}_1, \hat{a}_2)) \le \bar{v}_1.$$

The Perfect Folk Theorem implies that there exists δ^{***} so that for all $\delta \ge \delta^{***}$, any $\beta^*(h_t) \quad h_t \in G_1 \cup G_2$ can be supported by subgame-perfect Nash equilibrium.

I have shown that for all $\delta \ge \max\{\delta^*, \delta^{**}, \delta^{***}\}$ and $\mu_0 \le \bar{\mu}_0$, I can construct a perfect Bayesian equilibrium with the equilibrium payoff for the rational type of Player 1 equal to $v_1^1 \in [v_1 - (1-\delta)d, v_1]$. By a similar argument, I can construct another equilibrium in which the equilibrium payoff for the rational type of Player 1 equal to $v_1^2 \in [v_1, v_1 + (1-\delta)d] \in V_1^*$. An equilibrium with the payoff for the rational Player 1 exactly equal to v_1 can be constructed by taking a convex combination of the two equilibria. This can be achieved by making Player 2 mixes with an appropriate probability between the two equilibria in the first period. □

4.2 History-Dependent Commitment Strategies

In many games, including repeated Prisoners' Dilemma, a player may want to commit to a history-dependent strategy. When the commitment strategy is history independent, Player 2 pays only a one-period cost for not choosing a best response in any single period, as his current action does not the future behavior of the commitment type. The same is not true when the commitment type is history dependent. We can imagine a commitment strategy of "escalating punishment," which punishes Player 2 for not choosing a best response to the commitment strategy by switching to the minmax action for n periods, with n increasing with the number of times Player 2 has chosen a non-best-response in the past. Against such a commitment strategy, Player 2 will suffer a long-term loss for a single period of screening and, hence, may be less inclined to do so. A natural question is whether having more complicated commitment strategies would allow Player 1 to develop a reputation more effectively.

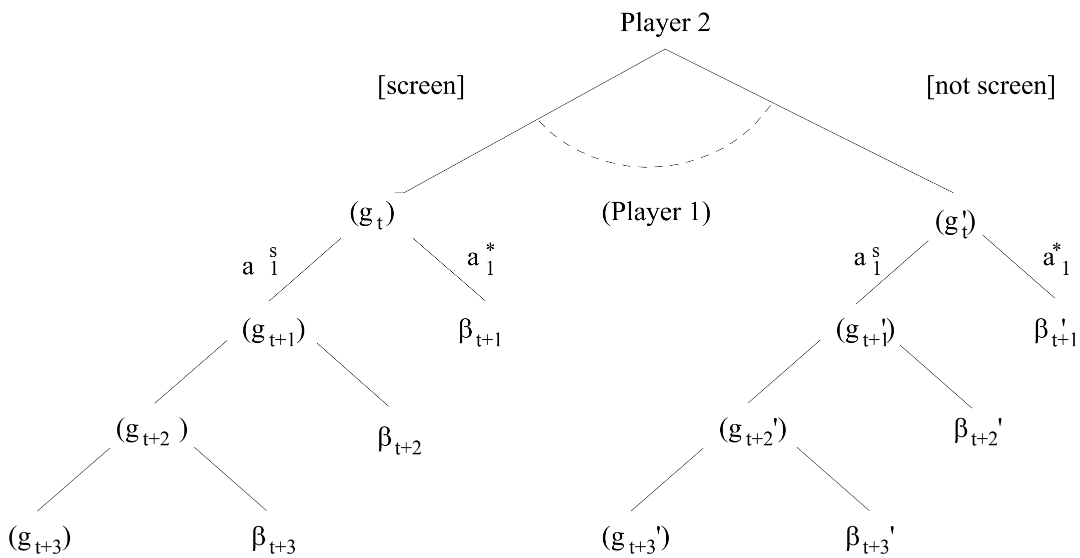

Figure 4: Player 2's decision in period t

Player 2 pays a cost for screening when Player 1 is a commitment type. For Player 2 to screen, he must be punished for not screening when Player 1 is rational. The issue is how to structure the punishment. In order to understand the problem, we need to consider the details of the equilibrium in Theorem 1. The decision tree in Fig. 4 represents the decision facing Player 2 in period t during the screening phase. Player 2 expects that the commitment action be chosen (by any type of Player 1) with probability $1-\Delta$ and a non-commitment action with probability Δ. If Player 2 chooses a_1^s, he will receive g_t in that period and a continuation payoff v_t , which takes the form:

$$\begin{aligned} v_t &= \delta[(1-\delta)(1-\Delta)g_{t+1}+\Delta\beta_{t+1}]+\delta^2(1-\Delta)[(1-\delta)(1-\Delta)g_{t+2}+\Delta\beta_{t+2}] \\ &\quad +\delta^3(1-\Delta)^2[(1-\delta)(1-\Delta)g_{t+3}+\Delta\beta_{t+3}]+.... \end{aligned}$$

where g_{t+i} is Player 2's expected stage-game payoff in period $t+i$ if Player 1 has not revealed that she is rational before that period, and β_{t+i} is the continuation payoff from period $t+i$ onward if Player 1 reveals that she is rational in period $t+i-1$.

On the other hand, if Player 2 deviates and chooses an action other than a_1^s, he receives g_t' and a continuation payoff v_t' given by

$$\begin{aligned} v_t' &= \delta[(1-\delta)(1-\Delta)g_{t+1}' + \Delta\beta_{t+1}'] + \delta^2(1-\Delta)[(1-\delta)(1-\Delta)g_{t+2}' + \Delta\beta_{t+2}'] \\ &\quad + \delta^3(1-\Delta)^2[(1-\delta)(1-\Delta)g_{t+3}' + \Delta\beta_{t+3}'] + ... \end{aligned}$$

where g_{t+i}' is Player 2's expected stage-game payoff in period $t+i$ if Player 1 has not revealed that she is rational before that period, and β_{t+i}' is the continuation payoff from period $t+i$ onward if Player 1 reveals that she is rational in period $t+i-1$.

When the commitment strategy is history independent, Player 2's actions do not affect the future actions of the commitment type. Furthermore, as the continuation strategy of Player 2 is by construction independent of Player 2's previous actions, Player 2's payoffs in periods where Player 1 has not been revealed as rational are independent of Player 2's previous actions, meaning that for all $s \geq t$, $g_{s+1} = g_{s+1}'$. Player 2 thus pays only a one-period loss equal to $(1-\delta)(g_t' - g_t)$ for screening. In that case, it is feasible to set $\beta_{s+2} = \beta_{s+2}'$ for all $s \geq t$ and punish Player 2 for not screening in a particular period only when Player 1 reveals that she is rational in the same period. As in the example in Section 2, it is rational for Player 2 to screen if

$$(1-\delta)(g_t' - g_t) \leq \Delta\delta(\beta_{t+1} - \beta_{t+1}')$$

The relation puts a lower bound on Δ and, hence, on the rate of Bayesian updating. To ensure that screening can last for many periods, it is crucial that the cost of screening in any period is on the order of $(1-\delta)d$. When the commitment strategy is history dependent, Player 2's action in period t can affect the commitment type's future actions. That is, g_{s+1} need not equal g_{s+1}'. Hence, the equilibrium construction in the last section would not work.

Theorem 2 shows that the conclusion of Theorem 1 still applies for a wide class of history-dependent commitment strategies. In the equilibrium in the proof below, Player 2's "punishment" for not screening is distributed over many periods in a way depending on how the action affects the commitment type's future actions. Roughly speaking, for all $s \geq t$, β_{s+i} and β_{s+i}' are set to satisfy the following equation:

$$[(1-\delta)(1-\Delta)g_{s+i} + \Delta\beta_{s+i}] = [(1-\delta)(1-\Delta)g_{s+i}' + \Delta\beta_{s+i}'],$$

so that the continuation payoffs v_1 and v_1' are equal to each other. That is, β_{s+i} and β_{s+i}' are chosen to offset the loss choosing the screening action in period t would cost period $s+i$.

The equilibrium construction in Theorem 2 requires an extra restriction on the commitment payoff. Let x denote Player 1's continuation payoff after she reveals her type. In equilibrium, x

must be set so that a rational Player 1 is indifferent between mimicking the commitment type and revealing her type. In order to set β_{s+i} and β'_{s+i} according to the equation above, I must ensure that both (x, β_{s+i}) and (x, β'_{s+i}) belong to V^*, which requires x be less than $\bar{v}_1$. As Player 1's continuation payoff is close to the commitment payoff near the end of the screening phase, the commitment payoff must also be strictly less than $\bar{v}_1$. See Figure 5.

Let $\bar{\nu}(\gamma_1) \equiv \sup\{\nu_\delta(\gamma_1|h_t)\ \forall h_t \in H, \delta \in (0,1)\}$ be the supremum of the set of continuation commitment payoffs (including the original commitment payoff at the beginning of the game) after any finite history and for any discount factor.

Assumption 1 $\bar{\nu}(\gamma_1) < \bar{v}_1$.

Theorem 7 *In a two-person infinitely-repeated game with incomplete information* $\Gamma(A, g, \delta, \gamma_1, \mu_0)$ *if* γ_1 *is a pure history-dependent strategy with* $\bar{\nu}(\gamma_1) < \bar{v}_1$*, then* $\forall v_1 \in V_1^*$*,* $\exists\ \underline{\delta}$ *and* $\bar{\mu}_0$*, such that* $\forall \delta \geq \underline{\delta}$ *and* $\mu_0 \leq \bar{\mu}_0$*, there is a perfect Bayesian equilibrium in which the average discounted payoff for the rational type of Player 1 is equal to* v_1*.*

Theorem 2 means that in two-person infinitely-repeated games with one-sided incomplete information, if the commitment strategy satisfies Assumption 1 and if the players are sufficiently patient and the prior probability that Player 1 is a commitment type is sufficiently small, then any $v_1 \in V_1^*$ can be supported by perfect Bayesian equilibrium. It implies, for example, in *infinitely-repeated* Prisoners' Dilemma, Player 1 may not receive the "cooperation" payoff by mimicking a "Tit for Tat" type.[17]

In games where $\bar{v}_1 \in V_1^*$ (e.g. Battle of Sexes), Assumption 1 directly rules out commitment types with commitment payoff equal to $\bar{v}_1$. In games where $\bar{v}_1 \notin V_1^*$ (e.g. Prisoners' Dilemma), where the commitment payoff is, by definition, strictly lower than $\bar{v}_1$, Assumption 1 requires that the commitment payoff be bounded away from $\bar{v}_1$ as δ approaches one. While Assumption 1 is not innocuous, it is not as restrictive as it might seem. The commitment types whose commitment payoff equals $\bar{v}_1$ are essentially simple commitment types, as they must choose a fixed stage-game action (the one that leads to the commitment payoff) most of the times. And, from Theorem 1, we know that reputation effects do not exist for simple commitment types.

Proof of Theorem 2:

Since $\bar{\nu}(\gamma_1) < \bar{v}_1$ and $v_1 > \underline{v}_1$, there exists $\hat{v}_1$, $\check{v}_1$, and $\epsilon > 0$ such that $\check{v}_1 \in (\underline{v}_1, v_1 - \epsilon]$ and

[17] Kreps, et al.(1982) show in *finitely-repeated* prisoners' dilemma, when there is some small probability that one of the players is a "Tit for Tat" type, then in all sequential equilibria both players will cooperate until almost the end of the game.

$\hat{v}_1 \in [\bar{\nu}(\gamma_1) + \epsilon, \bar{v}_1)$. Both $\check{v}_1$, $\hat{v}_1$ belong to V_1^* as V^* is convex and non-empty. There are $\check{v}_2$, $\hat{v}_2$ strictly larger than $\underline{v}_2$ so that $(\check{v}_1, \check{v}_2)$ and $(\hat{v}_1, \hat{v}_2)$ belong to $int(V^*)$. For convenience, I shall assume that the commitment payoff is always higher than $\check{v}_1$. Let $f : V_1 \to \Re$ be the linear function representing the line connecting $(\check{v}_1, \check{v}_2)$ and $(\hat{v}_1, \hat{v}_2)$. Since V^* is convex, there exists $q > 0$ such that the set $B \equiv \{(v_1, v_2) : v_1 \in [\check{v}_1, \hat{v}_1], v_2 \in [f(v_1) - q, f(v_1) + q]\} \subset V^*$. For any point (v_1, v_2), if $v_1 \in [\check{v}_1, \hat{v}_1]$ and if the vertical distance between (v_1, v_2) and $(v_1, f(v_1))$ is less than q, then (v_1, v_2) is strictly individually rational. See Figure 5.

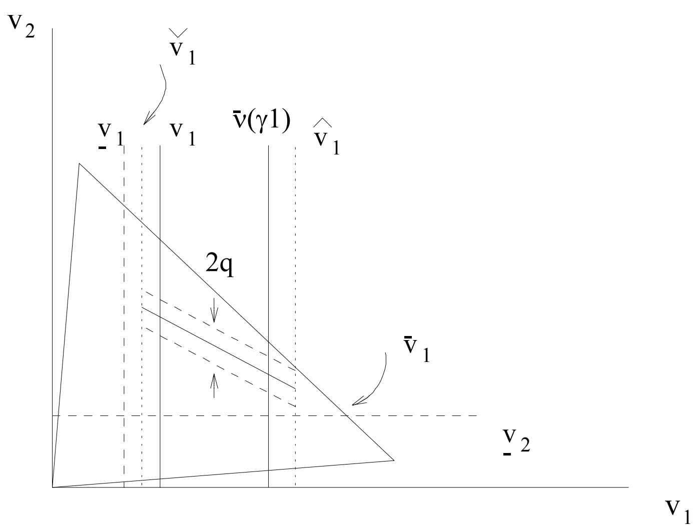

Figure 5: Stage-game Payoff for when $\bar{\nu}(\gamma_1) < \bar{v}_1$

For any action $a_1 \in A_1$, there is always an action a_2 such that $g_1(a_1, a_2) \leq \underline{v}_1$. Let $r : A_1 \to A_2$ be the function so that $g_1(a_1, r(a_1)) \leq \underline{v}_1$. Define $s_2^* : H(\gamma_1) \to A_2$ such that for all $h_t \in H(\gamma_1)$, $s_2^*(h_t) = r(\gamma_1(h_t))$. The function $s_2^*(h_t)$ will serve as a screening device. If Player 2 chooses $s_2^*(h_t)$, Player 1 will receive her minmax payoff at most for choosing $\gamma_1(h_t)$. Player 1's stage-game best response to $s_2^*(h_t)$ generally would depend on h_t. But for convenience I shall continue to use a_1^* to denote Player 1's best response to $s_2^*(h_t)$ among the non-commitment actions.

From now on, I assume that the players are sufficiently patient so that the players care little about the payoff in any single period. Specifically, I assume that $\frac{1-\delta}{\delta}$ is less than $\frac{\epsilon}{2d}$ and $\frac{q}{2d}$. For any $\delta \geq \delta^* = \min\{\frac{2d}{2d+\epsilon}, \frac{2d}{2d+q}\}$, define $\Lambda : H(\gamma_1) \to \Re$ such that:

$$\Lambda(h_t) = \begin{cases} v_1 & h_t = h_1 \\ \frac{1}{\delta^{t-1}}\{v_1 - (1-\delta)\sum_{i=2}^{t} \delta_1^{i-2} g_1(h_t^1(i-1), r(h_t^1(i-1)))\} & h_t \in H(\gamma_1),\ t \geq 2 \end{cases}$$

$\Lambda(h_t)$ will be the equilibrium continuation payoff for the rational type of Player 1 after h_t.[18] By construction, Player 1's expected payoff for the entire game is equal to v_1. Define the following subsets of finite histories:

$$\begin{aligned} H_1 &= \{h_t \in H(\gamma_1): \ \Lambda(h_t) < \nu(\gamma_1)\} \\ H_2 &= \{h_t.a \in H(\gamma_1): \ h_t \in H_1 \text{ and } \nu(\gamma_1) \leq \Lambda(h_t) \leq \hat{v}_1 - \frac{\epsilon}{2}\} \end{aligned}$$

For any h_{t+1}, h_t derived from the same history h consistent with γ_1,

$$\Lambda(h_t) + \frac{1-\delta}{\delta}(v_1 - \underline{v}_1) \leq \Lambda(h_{t+1}) \leq \Lambda(h_t) + \frac{1-\delta}{\delta}d.$$

meaning that $\Lambda(h_t)$ is strictly increasing in t (along h) and that there is some t^* such that $h_{t^*} \in H_2$. Furthermore, for all $h_t \in H_1 \cup H_2$, $v_1 \leq (1-\delta^t)\underline{v}_1 + \delta^t \hat{v}_1$; hence

$$t \leq \ln\left(\frac{v_1 - \underline{v}_1}{\hat{v}_1 - \underline{v}_1}\right) / \ln \delta.$$

Let T be the smallest integer larger than $\ln\left(\frac{v_1 - \underline{v}_1}{\hat{v}_1 - \underline{v}_1}\right) / \ln \delta$. For any history consistent with γ_1, the number of periods it takes to reach $H_2(\gamma_1)$ is uniformly bounded below T. In equilibrium, screening takes place only in periods with $h_t \in H_1 \cup H_2$. Thus, screening last for T periods at most. Lemma 2.3 implies that when the prior probability that Player 1 is a commitment type is sufficiently small, Player 2 will not be convinced that Player 1 is a commitment type in T periods. Define $\Delta \equiv \frac{(1-\delta)}{\delta}\frac{d}{q}$. Formally, there exists $\bar{\mu}_0, \delta^{**}$ such that for all $\mu_0 \leq \bar{\mu}_0$, for all $\delta \geq \delta^{**}$, and for all $t \leq T(\delta)$, $\frac{\mu_0}{(1-\Delta)^t} \leq \bar{\mu} = \frac{q}{2(d+q)}$.

For all $\mu_0 \leq \bar{\mu}_0$, and for all $\delta \geq \max\{\delta^*, \delta^{**}\}$, define σ_1^* and σ_2^* in $H(\gamma_1)$, and μ^* in H according to Tables 4 and 5.

In periods with $h_t \in H_1 \cup H_2$, Player 2 chooses s_2^*, and if a rational Player 1 imitates the commitment type, she will receive the minmax payoff $\underline{v}_1$at most.[19] When $h_t \in H_1$, a rational Player 1 chooses $\gamma(h_t)$ with probability $1 - \frac{\Delta}{1-\mu(h_t)}$ and a_1^* with probability $\frac{\Delta}{1-\mu(h_t)}$. The beliefs μ^* are defined so that it is consistent with Bayes' rule given σ_1^*. When $h_t \in H_2$, a rational Player 1 reveals her type by choosing a_1^*. .

A rational Player 1 can reveal her type under one of the following three situations: during the screening phase in H_1, in a final period of screening in H_2, or after the screening phase in

[18]Note that Λ defines continuation payoffs after all histories consistent with the commitment strategy, including the ones in which Player 2 deviates from his equilibrium strategy.

[19]She may receive less than $\underline{v}_1$, if she chooses a dominated action.

$\mathbf{h_t}$	$\boldsymbol{\sigma_1^*}(\mathbf{h_t})$	$\boldsymbol{\sigma_2^*}(\mathbf{h_t})$
H_1	$\sigma_1^*(a_1^*\|h_t) = \frac{\Delta}{1-\mu(h_t)}$ $\sigma_1^*(\gamma_1(h_t)\|h_t) = (1 - \frac{\Delta}{1-\mu(h_t)})$	$\sigma_2^*(s_2^*(h_t)\|h_t) = 1$
H_2	$\sigma_1^*(a_1^*\|h_t) = 1$	$\sigma_2^*(s_2^*(h_t)\|h_t) = 1$
$H(\gamma_1)/(H_1 \cup H_2)$	$\sigma_1^*(\gamma_1(h_t)\|h_t) = 1$	$\sigma_2^*(b_2(\gamma_1\|h_t)\|h_t) = 1$

Table 4: The equilibrium strategies, $\sigma^*(h_t) \quad \forall h_t \in H(\gamma_1)$

$\mathbf{h_t}$	$\boldsymbol{\mu}(\mathbf{h_t})$
$H_1 \cup H_2$	$\frac{\mu_0^1}{(1-\Delta)^{t-1}}$
$H/H(\gamma_1)$	0
$H(\gamma_1)/H_1 \cup H_2$	1

Table 5: Player 2's beliefs about Player 1's type

$H(\gamma_1)/(H_1 \cup H_2)$. The following sets of histories correspond to these situations.

$$\begin{aligned} G_3 &= \{h_t.a \in G(\gamma_1) : h_t \in H_1\} \\ G_4 &= \{h_t.a \in G(\gamma_1) : h_t \in H_2\} \\ G_5 &= \{h_t.a \in G(\gamma_1) : h_t \in H(\gamma_1)/(H_1 \cup H_2)\} \end{aligned}$$

The continuation payoffs β^* in each of these sets are given in Tables 6 and 7.

The equilibrium strategy and continuation payoffs are so constructed that for any $h_t \in H_1 \cup H_2$, the player's equilibrium continuation payoff after h_t, $v_1^*(h_t)$ and $v_2^*(h_t)$, are equal to $\Lambda(h_t)$ and $f(\Lambda(h_t))$, respectively.

It is straightforward to verify that σ_1^* is rational given σ_2^* and β^*. In $h_t \in H(\gamma_1)/H_1 \cup H_2$, the continuation commitment payoff, $\nu_\delta(\gamma_1|h_t)$, is by assumption larger than $(1-\delta)d + \delta\check{v}_1$, the highest payoff a rational Player 1 can obtain by deviating; it is thus optimal for her to follow the equilibrium strategy. The continuation payoffs β^* are defined so that in any period $h_t \in H_1 \cup H_2$, if a rational

$\mathbf{h_t}$	$\boldsymbol{\beta_1^*}(\mathbf{h_t})$	$\boldsymbol{\beta_2^*}(\mathbf{h_t})$
$h_t = h_{t-1}.a \in G_3 \cup G_4$	$\frac{\Lambda(h_{t-1})-(1-\delta)g_1(a_1^*, s_2^*(h_{t-1}))}{\delta}$	$f(\beta_1^*(h_t)) + \eta(h_t)$
$h_t = h_{t-1}.a \in G_5$	$\check{v}_1$	$f(\check{v}_1)$

Table 6: Continuation Payoffs $\beta^*(h_t) \quad \forall h_t \in G_3 \cup G_4 \cup G_5$

$\mathbf{h_t}$	$\boldsymbol{\eta}(\mathbf{h_t})$
$h_t = h_{t-1}.a \in G_3$	$\frac{q}{d}\{f(g_1(\rho_1^*(h_{t-1}), s_2^*(h_{t-1}))) - g_2(\rho_1^*(h_{t-1}), a_2)\}$
$h_t = h_{t-1}.a \in G_4$	$\frac{\mu(h_{t-1})}{1-\mu(h_{t-1})}\left\{\frac{f(\Lambda(h_{t-1}))-(1-\delta)f(g_1(\gamma_1(h_{t_1}), s_2^*(h_{t-1})))}{\delta} - v_2(\gamma_1\vert h_t, b_2(\gamma_1\vert h_t))\right\}+$ $\frac{1}{1-\mu(h_{t-1})}\frac{1-\delta}{\delta}\{f(g_1(\rho_1^*(h_{t-1}), s_2^*(h_{t-1}))) - g_2(\rho^*(h_{t-1}), a_2)\}$

Table 7: $\eta(h_t) \quad \forall h_t \in G_3 \cup G_4$

Player 1 reveals her type by choosing a_1^*, she will receive a payoff of $\Lambda(h_t)$. In periods with $h_t \in H_2$, Player 1 will strictly prefer to choose a_1^*, as $\Lambda(h_t)$ is strictly larger than the commitment payoff. In periods with $h_t \in H_1$, Player 1 is indifferent between $\gamma(h_t)$ and a_1^*, as Player 1's payoff for choosing the commitment action also equals $\Lambda(h_t)$.[20]

For all $h_t \in H_1 \cup H_2$, let $\pi_2^*(a_2, h_t)$ denote the payoff Player 2 will receive if he chooses a_2 in that period and the equilibrium strategy thereafter. I shall show that for all $h_t \in H_1 \cup H_2$ and for all $a_2 \in A_2$,

$$\pi_2^*(a_2, h_t) = f(\Lambda(h_t)). \tag{5}$$

That is, Player 2 is indifferent between following the equilibrium strategy and deviating once. Define $\{K_i\}_1^T$, a sequence of subsets of $H_1 \cup H_2$, as follows:

$$K_1 = H_2,$$
$$K_i = \{h_t \in H_1 : \forall a_2 \in A_2, \quad h_t.(\gamma_1(h_t), a_2) \in K_{i-1}\} \cup K_{i-1} \text{ for } 2 \leq i \leq T.$$

K_i includes those members of $H_1 \cup H_2$ which are no more than $i-1$ periods away from the last period of screening. As screening lasts T periods at most, $K_T = H_1 \cup H_2$.

[20] Recall that a_1^* is by definition the best response to the screening action among the non-commitment actions. Since the continuation payoff for the rational type of Player 1 after she has revealed her type does not depend on which non-commitment action she chose, Player 1 strictly prefers a_1^* to any other non-commitment action.

In any period with $h_t \in K_1$, if Player 2 chooses a_2 in that period and the equilibrium strategy thereafter, he will receive an expected payoff of $g_2(\rho_1^*(h_t), a_2)$ in that period and a continuation payoff of either $v_2(\gamma_1|h_{t+1}, b_2|h_{t+1})$ or $f(\beta_1^*(h'_{t+1})) + \eta(h'_{t+1})$, depending on the action of Player 1 in that period. Thus, $\pi_2^*(a_2, h_t)$ is given by

$$\pi_2^*(a_2,h_t) = (1-\delta)g_2(\rho_1^*(h_t),a_2)+\delta\{\mu(h_t)v_2(\gamma_1|h_{t+1},b_2(\gamma_1|h_{t+1}))+(1-\mu(h_t))(f(\beta_1^*(h'_{t+1}))+\eta(h'_{t+1}))\}$$

where $h_{t+1} = h_t.(\gamma_1(h_t), a_2)$ and $h'_{t+1} = h_t.(a_1^*, a_2)$. Substituting $\beta^*(h'_{t+1})$ and $\eta(h'_{t+1})$ out of the equation gives

$$\begin{aligned}
\pi_2^*(a_2,h_t) &= f(\Lambda(h_t)) - \delta\mu(h_t)\{(v_2(\gamma_1|h^*_{t+1},b_2(\gamma_1|h^*_{t+1})) - v_2(\gamma_1|h_{t+1},b_2(\gamma_1|h_{t+1}))\} \\
&\quad +(1-\delta)\{f(g_1(\rho_1^*(h_t),s_2^*(h_t)) - g_2(\rho^*(h_t),a_2))\} + (1-\mu(h_t))\delta\eta(h'_{t+1}) \\
&= f(\Lambda(h_t)).
\end{aligned}$$

where $h^*_{t+1} = h_t.(\gamma_1(h_t), s_2^*(h_t))$ and $h'_{t+1} = h_t.(a_1^*, s_2^*(h_t))$. The value of $\eta(h'_{t+1})$ is designed so that Player 2's expected payoff for choosing any $a_2 \in A_2$ is equal to $f(\Lambda(h_t))$.

Suppose Equation 5 holds for all $h_t \in K_i$. In a period $h_t \in K_{i+1}/K_i$, if Player 2 chooses a_2 and follows the equilibrium strategy thereafter, his expected payoff will be equal to

$$\begin{aligned}
\pi_2^*(a_2,h_t) &= (1-\delta)g_2(\rho_1^*(h_t),a_2) + \delta\{\Delta\beta_2^*(h_t.(a_1^*,a_2))] + (1-\Delta)f(\Lambda(h_t.(\gamma_1(h_t),a_2))\} \\
&= f(\Lambda(h_t)) + (1-\delta)\{g_2(\rho_1^*(h_t),a_2) - f(g_1(\rho_1^*(h_{t-1}),s_2^*(h_{t-1})))\} + \delta\Delta\eta(h_t.(a_1^*,a_2)) \\
&= f(\Lambda(h_t)).
\end{aligned}$$

As Equation (13) holds for all $h_t \in K_1$, by induction, it holds for all $h_t \in H_1$.

To complete the proof, I need to show that for all $h_t \in G_3 \cup G_4 \cup G_5$, the continuation payoff $\beta^*(h_t)$ belongs to V^*. As for all $h_t \in H_1 \cup H_2$, $\Lambda(h_t) \in [v_1, \hat{v}_1 - \frac{\epsilon}{2}]$, it follows from the definitions of $\check{v}_1$ and $\hat{v}_1$ that

$$\check{v}_1 \leq \beta_1(h_t) \leq \hat{v}_1 \quad h_t \in G_3 \cup G_4$$

By construction, the difference of $f(g_1(\rho_1^*(h_{t-1}), s_2^*(h_{t-1})))$ and $g_2(\rho_1^*(h_{t-1}), a_2)$ is less than d; therefore, for all $\forall h_t \in G_3$, $|\eta(h_t)| \leq q$. For all $h_t \in G_4$, as $\mu(h_t) \leq \frac{q}{2(d+q)}$, and $\delta \geq \frac{2d}{2d+q}$,

$$|\eta(h_t)| < \frac{\mu(h_{t_1})}{1-\mu(h_{t-1})}d + \frac{1}{1-\mu(h_{t-1})}\frac{1-\delta}{\delta}d \leq q.$$

By construction, $(v_1, v_2) \in V^*$ if $v_1 \in [\check{v}_1, \hat{v}_1]$ and $v_2 \in [f(v_1) - q, f(v_1) + q]$. Thus, for all $h_t \in G_3 \cup G_4 \cup G_5$, $\beta^*(h_t) \in V^*$. The folk theorem implies that there exists a δ^{***}, such that $\forall \delta \geq \delta^{***}$, β^* can supported by Subgame Perfect Nash Equilibrium. □

4.3 Concluding Remarks

Although Theorems 1 and 2 formally assume that the players are equally patient and that there is only one pure commitment type, these assumptions are not crucial to my results. Below, I briefly discuss how my results extend to more general cases.

- To extend the results to games with multiple commitment types, I need to construct an equilibrium in which Player 1 mimics all commitment types with strictly positive probability during the screening phase. As Player 2 can distinguish between two possible pure commitment types once they have chosen different actions, I need only to ensure that in information sets in which the commitment types choose different actions, the continuation payoffs for the rational type of Player 1 are set so that she is willing to mix between all commitment actions.

- The conclusion of Theorem 2 will continue to hold when mixed commitment strategies are allowed, if an additional assumption is made to guarantee that a rational Player 1 can always reveal her type.[21] The problem becomes complicated in this case as the definition of commitment payoff needs to be generalized and the equilibrium strategies of the rational type of Player 1 and Player 2 will involve mixed actions.[22] Nonetheless, the basic idea of the proof is the same as that of Theorem 2.

- The Perfect Folk Theorem without public randomizations has not been proven in the case where the players have different discount factors[23] To extend the results to the case of comparably patient players, I need to assume the availability of public randomizations.

5 Reputation Effects in Strongly-Dominant-Action Games

In this section I show that in strongly-dominant-action games, a player can build a reputation even when she is less patient than her opponent. Suppose $y < 0$ and $x > 0$ in the stage game depicted in Fig.6. In this game, the best outcome for Player 1 is (A, A), which gives her a payoff of 2. If Player 1 is rational, she always chooses A, as A is strictly dominant. If Player 2 is

[21]Formally, this is equivalent to assuming that the commitment strategies do not have full support at any information set.

[22]However,in this case it is unnecessary to deal with the issue of multiple commitment types explicitly because from the perspective of Player 2, he is always facing one "aggregate" commitment type with mixed commitment strategies. For example, suppose there are two commitment types: ω_1 with probability p_1 and ω_2 with probability p_2; for Player 2, this is equivalent to facing a commitment type $\gamma_1 = \frac{p_1}{p_1+p_2}\omega_1 + \frac{p_2}{p_1+P_2}\omega_2$ with a probability of $p_1 + p_2$.

[23]Lehrer and Pauzner (1997) has shown that the folk theorem holds with different discount factors when mixed strategies are observable.

		Player 2	
		A	B
Player 1	A	2,x	0,0
	B	0,0	y,2

Figure 6: A Strongly-Dominant-Action Game

rational and knows that Player 1 is rational, then he should also choose A. Thus, if the game is played once, the likely outcome is (A, A). If the game is repeated and the players are sufficiently patient, then, according to the folk theorem, any payoff profile strictly larger than $(0, \frac{2x}{2+x})$ can be supported by subgame-perfect Nash equilibrium. Theorem 4 below, however, shows that the folk theorem is not robust in this example. If there is any probability that Player 1 is a commitment type always choosing A, then (A, A) will be chosen on the equilibrium path of any perfect Bayesian equilibrium in the incomplete-information infinitely-repeated version of the game. The result hinges on the assumptions that there is only one commitment type and that the commitment type always chooses A, but it does not require that Player 1 be infinitely more patient than Player 2. In fact, the result holds even when Player 1 is less patient than Player 2.

The intuition is as follows. Suppose, by way of contradiction, that there exists an equilibrium in which Player 1's payoff is strictly less than 2 and in which (A, A) is not always chosen on equilibrium path. That Player 1's payoff is strictly less than 2 means that if Player 1 deviates and chooses A indefinitely, Player 2, according to the equilibrium, must eventually choose B with positive probability. Since B is not a best response for Player 2 against A, Player 2 chooses B only when he believes that Player 1 may choose B either in that period or in the future. But it is rational for Player 1 to choose B only if she believes that Player 2 may choose B in the future, as A is strictly dominant. In short, Player 2 chooses B because he thinks Player 1 may choose B in the future, and Player 1 chooses B because she also thinks Player 2 may choose B in the future, and so on.[24] The process, however, cannot last indefinitely when Player 2 rationally updates his belief about Player 1's type. Eventually, either Players 1 or 2 must play sub-optimally in some period, in contradiction to the equilibrium assumption. Theorem 3 formalizes this argument.

Note that when $x > 2 > y > 0$, the game becomes a common-interest game, but A is no longer a strictly dominant action for Player 1. Since there is no conflict of interests between the players, it may appear that this would be a situation particularly conducive to reputation development. My

[24]Note that it is important here that Player 1 expects Player 2 to choose B strictly in the future.

results show that common interest is neither necessary nor sufficient for the existence of reputation effects. Theorem 1 implies that when A is not strictly dominant, even if there is a small probability that Player 1 is a commitment type always choosing A, there still exists perfect Bayesian equilibrium in which Player 1's equilibrium payoff is arbitrarily close to her minmax payoff.[25] On the other hand, it does not matter whether the game is of common interest, (i.e. $x > 2$), because Player 2 will choose A whenever he is convinced that Player 1 is going to choose A, regardless of the value of x.

Theorem 8 *In an infinitely-repeated strongly-dominant-action game with one-sided incomplete information, $\Gamma(A, g, \delta_1, \delta_2, \gamma_1, \mu_0^1)$, if the commitment type, γ_1, always chooses the strongly-dominant action, then for any $\delta_1, \delta_2 \in (0,1)$, and any $\mu_0^1 > 0$, the equilibrium payoff for the rational type of Player 1 is equal to $\bar{v}_1$ in any perfect Bayesian equilibrium.*

The following three lemmas are useful in the proof.[26] Let $\hat{a}_1$ be the strongly-dominant action and $\hat{a}_2$ be a best response to $\hat{a}_1$. Let $\hat{v}_2$ denote $g_2(\hat{a}_1, \hat{a}_2)$.

Lemma 9 *If $(\sigma_1, \sigma_2, \mu)$ is a perfect Bayesian equilibrium, and $v_1(\gamma_1, \sigma_2|\hat{a}_1) = \bar{v}_1$, then $\sigma_1(h_1) = \hat{a}_1$.*

Lemma 5.1 follows as $\hat{a}_1$ is strictly dominant. It says that Player 1 always chooses $\hat{a}_1$ when doing so does not affect her continuation payoff. Lemma 5.2 says that Player 2 cares very little about his distant future payoffs. Formally, for any $\sigma_i \in \Sigma_i$ and positive integer N, define

$$\begin{aligned} \sigma_i^N(h_t) &= \sigma_i(h_t) \qquad \forall t \leq N-1 \\ \sigma_i^N(h_t) &= a_1' \qquad \forall t > N-1 \quad \text{for some } a_1' \in A_1. \end{aligned}$$

Lemma 10 *Given δ_2, $\forall \epsilon > 0$, $\exists N$ such that $\forall \sigma_1, \sigma_2$, $|v_2(\sigma_1, \sigma_2) - v_2(\sigma_1^N, \sigma_2^N)| \leq \epsilon$.*

Let $\rho_1 = \mu\gamma_1 + (1-\mu)\sigma_1$. Define $d^N(\rho_1) = \max_{s_2 \in S_2}\{1 - P_{\rho_1, s_2}(h_N(\gamma_1, s_2))\}$. Recall that $h_N(\gamma_1, s_2)$ is the $(N-1)$-period history induced by γ_1 and s_2, and $P_{\rho_1, s_2}(h_N(\gamma_1, s_2))$ is the probability that $h_N(\gamma_1, s_2)$ occurs when the rational Player 1 chooses σ_1 and Player 2 chooses s_2. The function $d^N(\rho_1)$ measures the "distance" between ρ_1 and γ_1.[27] Lemma 5.3 says that when ρ_1 and γ_1 are close, they give Player 2 similar payoffs.

[25] Cripps and Thomas (1997) first proves this result in the context of common-interest games. In their paper, they assume that A is not a strictly dominant action. Aumann and Sorin (1989) show that there exists a pure strategy equilibrium in this game, and all pure strategy equilibria are Pareto efficient.

[26] The proofs of the lemmas are given in the appendix.

[27] $d^N(.)$ is consistent with the definition of distance in Kalai and Lehrer (1993).

Lemma 11 *Given δ_2, $\forall\ \xi > 0$ and positive integer N, $\exists\ \epsilon > 0$ such that if $d^N(\rho_1) \leq \epsilon$, then $\forall \sigma_2 \in \Sigma_2$,*

$$|v_2(\rho_1^N, \sigma_2^N) - v_2(\gamma_1^N, \sigma_2^N)| \leq \xi.$$

Proof of Theorem 3:

Let $(\sigma_1, \sigma_2, \mu)$ be a perfect Bayesian equilibrium in which Player 1's equilibrium payoff is strictly below $\bar{v}_1$. I shall prove that such an equilibrium could not exist by showing that if it does, it will be possible to construct a sequence of periods $\{t_1, t_2, t_3, ...\}$ and a sequence of histories $\{\hat{h}_{t_1}, \hat{h}_{t_2}, ...\}$such that for all i and for some $\epsilon > 0$, $\mu(\hat{h}_{t_{i+1}}) \geq \frac{\mu(\hat{h}_{t_i})}{1-\epsilon}$, contradicting the requirement that $\mu(\hat{h}_{t_j})$ be not larger than one for all t_j.

Let $A_2' = \{a_2 \in A_2 : g_1(\hat{a}_1, a_2) < \bar{v}_1)$ be the set of Player 2's action that gives a Player 1 choosing $\hat{a}_1$a payoff below $\bar{v}_1$. By definition, any $a_2 \in A_2'$ is not a best response to $\hat{a}_1$. In equilibrium, the rational type of Player 1 is supposed to play σ_1. If she deviates and plays the commitment strategy γ_1, her payoff $v_1(\gamma_1, \sigma_2)$ must be lower than $\bar{v}_1$. Thus, σ_2 must dictate that Player 2, in response to Player 1 choosing γ_1, choose some $a_2 \in A_2'$ with positive probability. Formally, it means that there exists some history $h_t \in H(\gamma_1)$ such that $\sigma_2(\hat{a}_2|h_t) < 1$. Set $t_1 = t$ and $\hat{h}_{t_1} = h_t$. Starting at h_t, let s_2 be the pure repeated-game strategy in the support of $\sigma_2|h_t$ that chooses some $a_2 \in A_2'$ in the first period.[28] If Player 1 chooses γ_1 and Player 2 chooses s_2, Player 2 will receive an average payoff

$$v_2(\gamma_1, s_2) \leq \hat{v}_2 - c,$$

where $c = (1 - \delta_2)(\hat{v}_2 - \max_{a_2 \in A_2/br(\hat{a}_1)} g_2(\hat{a}_1, a_2))$ is the minimum loss for choosing a non-best-response against γ_1.

That Player 2 is willing to choose s_2, a non-best-response to γ_1, implies that the rational type of Player 1 will choose a non-commitment action with positive probability bounded away from zero in the future. Let $\rho_1' = \rho_1|h_t$. By Lemma 5.2, there is a N such that $v_2(\gamma_1^N, s_2^N) \leq \hat{v}_2 - \frac{7c}{8}$. By Lemma 5.3, there is a ϵ such that if $d^N(\rho_1', \gamma_1) \leq \epsilon$, then for all $\tilde{\sigma}_2 \in \Sigma_2$ $|v_2(\rho_1'^N, \tilde{\sigma}_2^N) - v_2(\gamma_1^N, \tilde{\sigma}_2^N)| \leq \frac{c}{8}$. I shall show that it is impossible for $d^N(\rho_1', \gamma_1) \leq \epsilon$. Suppose, to the contrary, that $d^N(\rho_1', \gamma_1) \leq \epsilon$, then

$$\begin{aligned} v_2(\rho_1'^N, s_2^N) &\leq v_2(\gamma_1^N, s_2^N) + \frac{c}{8} \\ &\leq \hat{v}_2 - \frac{6c}{8}. \end{aligned}$$

[28]Note that I start counting from h_t, so that the first period means the period with h_t and N periods means N periods after h_t.

Define $\hat{s}_2$ as the simple strategy of always choosing $\hat{a}_2$. Since s_2 is a best response against ρ_1,

$$v_2(\rho_1', s_2) \geq v_2(\rho_1', \hat{s}_2),$$

which implies that

$$\begin{aligned} v_2(\rho_1'^N, \hat{s}_2^N) &\leq v_2(\rho_1'^N, s_2^N) - \frac{c}{8} \\ &\leq \hat{v}_2 - \frac{5c}{8}. \end{aligned}$$

Since $v_2(\gamma_1^N, \hat{s}_2^N) \geq \hat{v}_2 - \frac{c}{8}$,

$$|v_2(\rho_1'^N, \hat{s}_2^N) - v_2(\gamma_1^N, \hat{s}_2^N)| \geq \frac{c}{2},$$

contradicting the assumption that $d^N(\rho_1', \gamma_1) \leq \epsilon$. Intuitively, in order to induce Player 2 to choose a non-best-response to γ_1, a rational Player 1 must deviate from γ_1 with probability bounded away from zero either when Player 2 chooses s_2 or $\hat{s}_2$. Let $\tilde{s}_2 \in S_2$ be the strategy that induces Player 1 to deviate from γ_1 with probability not less than ε. This means that $1 - P_{\rho_1', \tilde{s}_2}(h_N(\gamma_1, \tilde{s}_2)) \geq \epsilon$.

Along the history $h_N(\gamma_1, \tilde{s}_2)$, there is some final period n after which the rational Player 1 chooses $\hat{a}_1$ with probability one. For Player 1 to choose some action $a_1 \neq \hat{a}_1$ in period n, Player 2 must choose some $a_2 \in A_2'$ with positive probability in some future period. Note that it would not be sufficient for Player 2 to choose a non-best-response in the same period n as $\hat{a}_1$ is strictly dominant against all $a_2 \in A_2'$. Starting from period $t_1 + n$, there exists a t'-period history $h_{t'} \in H(\gamma_1)$, $t' > 1$, such that Player 2 may choose some action $a_2 \neq \hat{a}_2$. Set $t_2 = t_1 + n + t' - 1$ and $\hat{h}_{t_2} = \hat{h}_{t_1}.h_n(\gamma_1, \tilde{s}_2).h_{t'}$. Since $\hat{h}_{t_1}$, $h_n(\gamma_1, \tilde{s}_2)$, and $h_{t'}$ all belong to $H(\gamma_1)$, $\hat{h}_{t_2}$ belongs to $H(\gamma_1)$ as well. According to Bayes' rule, $\mu(\hat{h}_{t_2}) \geq \frac{\mu(\hat{h}_{t_1})}{1-\epsilon}$. Repeating the same argument, $t_3, t_4, t_5 ...$ and $\hat{h}_{t_1}, \hat{h}_{t_2}, ...$ can be constructed so that for all i, $\mu(\hat{h}_{t_{i+1}}) \geq \frac{\mu(\hat{h}_{t_i})}{1-\epsilon}$. For any $\mu_0^1 > 0$, there is a t_k such that $\mu(\hat{h}_{t_k}) > 1$. Thus, $(\sigma_1, \sigma_2, \mu)$ is not a perfect Bayesian equilibrium. □

6 Folk Theorems with Commitment Types

In this section, I strengthen the results of Section 4 in two ways. Theorem 4 is a folk theorem with one-sided incomplete information. It shows that for any two-person infinitely-repeated game with one-sided incomplete information which satisfies the conditions of either Theorems 1 or 2, any payoff profile in the interior of the convex hull of the strictly-individually-rational-payoff set can be supported by perfect Bayesian equilibrium, when the players are sufficiently patient and the prior probability that the informed player is a commitment type is sufficiently small. In other words, we can construct an equilibrium which gives a rational Player 1 a low payoff without harming Player 2.

Theorem 12 *If a two-person infinitely-repeated game with one-sided incomplete information* $\Gamma(A, g, \delta, \gamma_1, \mu_0^1)$ *satisfies the conditions of either Theorem 1 or 2, then* $\forall$ $(v_1, v_2) \in int(V^*)$, $\exists$ $\underline{\delta}$ *and* $\bar{\mu}_0^1$, *such that* $\forall \delta \geq \underline{\delta}$ *and* $\mu_0^1 \leq \bar{\mu}_0^1$, *there is a perfect Bayesian equilibrium with the rational players' equilibrium payoffs equal to* (v_1, v_2).

The idea behind Theorem 4 is very simple: Theorems 1 and 2 imply that there exists a perfect Bayesian equilibrium in which the equilibrium payoff for Player 1 is v_1; now we want the payoff for Player 2 to be equal to v_2. When it common knowledge that Player 1 is rational, the folk theorem implies that there exists a subgame-perfect Nash equilibrium in which the equilibrium payoffs for Players 1 and 2 are v_1 and v_2. Based on these two equilibria, I can construct a third one. In the third equilibrium, if Player 1 chooses the commitment action in the first period, the players plays the first equilibrium from period 2 onward; otherwise, they plays the second one. Since the rational type of Player 1 is indifferent between these two equilibria, it is rational for her to mix. The equilibrium payoff for Player 2 can be made approximately equal to v_2 by having the rational Player 1 to reveal her type with a high probability.

Proof of Theorem 4:

In period 1, a rational Player 1 chooses a_1^* with probability $p_1$and $\gamma_1(h_1)$ with probability $1 - p_1$. Player 2 chooses $\tilde{a_2}$, a best response to ρ_1, the expected strategy of Player 1 in period 1. For all $a_1 \in A_1$ and for all $a_2 \in A_2$, define $\Lambda(a_1, a_2) = \frac{v_1 - (1-\delta) g_1(a_1, \tilde{a_2})}{\delta}$.

The continuation payoffs for Players 1 and 2 from period 2 onward, denoted by $(v_1(a), v_2(a))$, and Player 2's period 2 belief about Player 1's type, denoted by $\mu(a)$, are given in Table 8.

a	$\mathbf{v_1(a)}$	$\mathbf{v_2(a)}$	$\boldsymbol{\mu}\mathbf{(a)}$
$a_1 = \gamma_1(h_1)$	$\Lambda(a)$	$f(\Lambda(a))$	$\frac{\mu_0^1}{\mu_0^1 + (1-\mu_0^1)(1-p_1)}$
$a_1 \neq \gamma_1(h_t)$	$\Lambda(a)$	$\frac{v_2 - (1-\delta) g_2(\rho_1, \tilde{a_2}) - (1-p_1+\mu_0^1 p_1)\delta f(\Lambda(a))}{(1-\mu_0^1) p_1}$	0

Table 8: The continuation payoffs, $(v_1(a), v_2(a))$, and beliefs , $\mu(a)$, in the second period

It is straightforward to verify that $\mu(a)$ is consistent with Bayes' rule, that given $(v_1(a), v_2(a))$ the first-period strategies of the players are rational, and that the equilibrium payoffs for the entire game are equal to (v_1, v_2). To complete the proof, I need only to show that the continuation payoffs can be supported by some perfect Bayesian equilibrium when δ_1 is sufficiently close enough to one and when μ_0^1 is sufficiently small.

Since $(v_1, v_2) \in int(V^*)$, there exist δ^* and μ^* such that for all $\delta \geq \delta^*$ and for all $\mu_0^1 \leq \mu^*$, p_1 can be set so that $(v_1(a), v_2(a)) \in V^*$ for all $a \in A$. When $a_1 \neq \gamma_1(h_t)$, the folk theorem implies that there exists δ_1^{**} such that for all $\delta_1 \geq \delta_1^{**}$, $(v_1(a), v_2(a))$ can be supported by subgame-perfect Nash equilibrium. When $a_1 = \gamma_1(h_1)$, Theorem 1 (or 2) implies that, in the game $\Gamma(A, g, \delta_1, \delta_2, \gamma_1|a, \mu(a))$, there exist δ^{***} and μ^{1**} such that for all $\delta \geq \delta^{***}$ and for all $\mu_0^1 \leq \mu^{1**}$, there exists perfect Bayesian equilibrium in which the equilibrium payoffs of Players 1 and 2 are equal to $\Lambda(a)$ and $f(\Lambda(a))$, respectively. Set $\mu_0^{1***} = \frac{(1-p_1)\mu^{1**}}{(1-p_1\mu^{1**})}$. It is straightforward to verify that Theorem 4 holds for all $\delta \geq \max\{\delta^*, \delta^{**}, \delta^{***}\}$ and for all $\mu_0^1 \leq \min\{\mu^{1*}, \mu^{1***}\}$. □

Theorem 5 extends Theorem 4 to the situation where both players possess private information. Formally, denote a two-person infinitely-repeated game with two-sided incomplete information by $\Gamma(A, g, \delta, \gamma_1, \mu_0^1, \gamma_2, \mu_0^2)$, where γ_2 represents the commitment type of Player 2 and μ_0^2 represents the prior probability that Player 2 is a commitment type. Let Γ_i denote the one-sided incomplete-information game in which only Player i possesses private information (i.e. $\mu_0^j = 0$, $j \neq i$). Theorem 5 states that in any infinitely-repeated game $\Gamma(A, g, \delta, \gamma_1, \mu_0^1, \gamma_2, \mu_0^2)$, if Γ_1 and Γ_2 satisfy the conditions of Theorems 1 or 2, then any payoff profile that is in the interior of V^* can be supported by a perfect Bayesian equilibrium when the players are sufficiently patient and the prior probability that the players are commitment types are sufficiently small.

The proof is similar to the one of Theorem 4. The key is to construct an equilibrium in which the rational types of both players reveal their types with high probability in the first period. In equilibrium, when each player expects the other player to reveal her type, each of them is basically choosing between a complete-information continuation game or a one-sided incomplete-information one. From Theorem 4, we know that there exist continuation equilibria in which the rational players are indifferent between the two choices. If the rational players choose to reveal their types with high probability in the first period, then their equilibrium payoff will mainly depend on their continuation payoffs in the complete-information continuation game.

Theorem 13 *In a two-person infinitely-repeated game with two-sided incomplete information* $\Gamma(A, g, \delta, \gamma_1, \mu_0^1, \gamma_2, \mu_0^2)$*, if both* Γ_1 *and* Γ_2 *satisfy the conditions of either Theorems 1 or 2, then*

$\forall(v_1, v_2) \in int(V^*)$, $\exists\ \underline{\delta}$, $\bar{\mu}_0^1$, and $\bar{\mu}_0^2$ such that $\forall \delta \geq \underline{\delta}$, $\mu_0^1 \leq \bar{\mu}_0^1$, and $\mu_0^2 \leq \bar{\mu}_0^2$, there is a perfect Bayesian equilibrium with the rational players' equilibrium payoffs equal to (v_1, v_2).

Proof of Theorem 5: In period 1, a rational Player i chooses a_i^* with probability p_i and $\gamma_i(h_i)$ with probability $1 - p_i$. Consider Player 1's decision. If she chooses a_1^* in period 1, she will play either a complete-information game or an one-sided incomplete-information game in which she is uncertain about Player 2's type in period 2. On the other hand, if Player 1 chooses $\gamma_1(h_1)$ in period 1, then she will play either an one-sided incomplete-information game in which Player 2 is uncertain about her type or a two-sided incomplete-information game. The situation is similar for Player 2. For all $a_1 \in A_1$ and for all $a_2 \in A_2$, define

$$\Lambda_1(a_1, a_2) = \frac{v_1 - (1-\delta)g_1(a_1, \rho_2)}{\delta_1} \text{and } \Lambda_2(a_1, a_2) = \frac{v_2 - (1-\delta)g_2(\rho_1, a_2)}{\delta_2}.$$

The continuation payoffs and the beliefs in period 2 are given in Table 9. The continuation

$\mathbf{a_1, a_2}$	$\mathbf{v_1(a)}$	$\mathbf{v_2(a)}$	$\boldsymbol{\mu_1}\mathbf{(a)}$	$\boldsymbol{\mu_2}\mathbf{(a)}$
$a_1 \neq \gamma_1(h_1)$ $a_2 \neq \gamma_2(h_1)$	$\Lambda_1(a)$	$\Lambda_2(a)$	0	0
$a_1 = \gamma_1(h_1)$ $a_2 \neq \gamma_2(h_1)$	$\frac{\Lambda_1(a)-(1-p_2+\mu_0^2 p_2)\tilde{v_1}}{(1-\mu_0^2)p_2}$	$\Lambda_2(a)$	$\frac{\mu_0^1}{\mu_0^1+(1-\mu_0^1)(1-p_1)}$	0
$a_1 \neq \gamma_1(h_t)$ $a_2 = \gamma_2(h_1)$	$\Lambda_1(a)$	$\frac{\Lambda_2(a)-(1-p_1+\mu_0^1 p_1)\tilde{v_2}}{(1-\mu_0^1)p_1}$	0	$\frac{\mu_0^2}{\mu_0^2+(1-\mu_0^2)(1-p_2)}$
$a_1 = \gamma_1(h_1)$ $a_2 = \gamma_2(h_1)$	$\tilde{v_1}$	$\tilde{v_2}$	$\frac{\mu_0^1}{\mu_0^1+(1-\mu_0^1)(1-p_1)}$	$\frac{\mu_0^2}{\mu_0^2+(1-\mu_0^2)(1-p_2)}$

Table 9: Continuation Payoffs, $v_1(a), v_2(a)$ in period 2

payoffs are set so that the equilibrium payoffs for the rational players are equal to (v_1, v_2). The exact value of $(\tilde{v}_1, \tilde{v}_2)$, the equilibrium payoffs in the two-sided incomplete-information subgame, does not matter.[29] It is straightforward to verify that $\mu_1(a)$ and $\mu_2(a)$ are consistent with Bayes' rule, and that the first period strategies are rational given the beliefs and continuation payoffs. Using an argument similar to that of Theorem 4, it can be shown that there exist $\underline{\delta}$, $\bar{\mu}_0^1$, and $\bar{\mu}_0^2$

[29] It is straightforward to see that equilibrium exists. By definition, $\exists v_2 \in V_2^*$ s.t. $(\nu(\gamma_1), v_2) \in V^*$. It is possible to construct an equilibrium with the following features: In period 1 the rational type of Player 1 reveals her type and the rational type of Player 2 chooses the commitment action. In period 2, from Theorem 4, construct a perfect Bayesian equilibrium for the one-sided incomplete-information continuation game with payoffs such that the payoff for the entire game is equal to $(\nu(\gamma_1), v_2)$. Player 1 is willing to reveal her type in period 1 as she is getting her commitment payoff in equilibrium.

such that for all $\delta \geq \underline{\delta}$, for all $\mu_0^1 \leq \bar{\mu}_0^1$, and for all $\mu_0^2 \leq \bar{\mu}_0^2$, so that $v_1(a)$ and $v_2(a)$ are supported by perfect Bayesian equilibrium. □

7 Conclusion

In this paper, I study a two-person infinitely-repeated game in which an informed player may try to develop a reputation by mimicking a commitment type, but her opponent, expecting that, tries to screen her out by choosing an action that will "punish" her for imitating the commitment type. Fudenberg and Levine (1989) and Schmidt (1993) point out that since screening is also costly to the uninformed player, he will screen only when he expects the informed player to reveal her type in the future with a probability bounded away from zero. When he sees the informed player choose the commitment action in a period in which he thinks that a rational type may choose a non-commitment action, he assigns a higher probability to the event that the informed player really is a commitment type. Based on this argument, they conclude that for any fixed discount factor of the uninformed player, the number of screening periods must be bounded. Hence, no matter how small the prior probability that the informed player is a commitment type, if she is sufficiently more patient than the uninformed player, she can guarantee herself a payoff strictly higher than the minimum payoff she may otherwise receive in the case of complete information. This result, while very robust in many other aspects, obviously depends heavily on the assumption that the informed player is arbitrarily more patient than her opponent.

The main contribution of this paper is to show that when the two players are comparably patient, the informed player does not always benefit from developing a reputation. Formally, when the commitment strategy is simple and the stage game is neither a strongly-conflicting-interest game nor a strongly-dominant-action game, or when the commitment payoff is strictly less than the informed player's highest repeated-game payoff, any payoff that is in the interior of the set of strictly individually rational payoffs can be supported by some perfect Bayesian equilibrium as the discount factor goes to one and the prior probability that the informed player is a commitment type goes to zero. This result is very robust. With minor qualifications, it can be extended to allow for multiple commitment types, mixed commitment strategies, comparably patient players and two-sided private information. In other words, the folk theorem holds even when there is a small probability that the players are commitment types.

The main idea is that the strength of reputation effects depend on the level of relative, but not absolute, patience. To clarify the relationship between reputation effects and relative patience, I introduce a measure of relative patience based on the weights the players put on their future payoffs. Absolute patience does not matter because as the informed player becomes more patient, the uninformed player will also become more patient and, hence, may screen for a longer period of time. For a fixed level of relative patience, if the prior probability that the informed player is a commitment type is sufficiently small, the uninformed player can be induced to screen for such a long period that it is not worthwhile for the informed player to develop a reputation.

It is crucial for the proofs of Theorem 1 and 2 that a rational informed player can be induced to mix between mimicking the commitment type and revealing her type. Giving up her reputation is not necessarily bad for a rational informed player as any strictly-individually-rational payoff can be supported by subgame-perfect Nash equilibrium. As a result, with the exception of strongly-dominant-action games, there always exist continuation equilibrium payoffs such that the rational informed player is indifferent between mimicking the commitment type and revealing her type. The importance of this point is demonstrated by the example of infinitely-repeated strongly-dominant-action games. In such games, if the only commitment type is one who always chooses the strongly-dominant action, the rational informed player cannot be induced to reveal her type in the last period of screening. In that case, I show that the informed player receives the commitment payoff in all perfect Bayesian equilibria. To the best of my knowledge, this is the only class of games in which the folk theorem is not robust.

8 Appendix

Proof of Lemma 2.1:

Suppose (1) is true, given any $\epsilon, \eta > 0$, $\exists\ \underline{\delta}_1$ s.t. $\forall \delta_1 \geq \underline{\delta}_1$

$$0 \leq \frac{\ln \delta_1}{\ln \delta_2(\delta_1)} \leq \frac{\ln(1-\eta)}{\ln \epsilon}$$

Since the last expression can be made to be arbitrarily small by pushing η to zero, $\lim_{\delta_1 \to 1} m(\delta_1, \delta_2(\delta_1))$ exists and is equal to 0.

To show the converse is true, suppose $\lim_{\delta_1 \to 1} m(\delta_1, \delta_2(\delta_1)) = 0$, then

$$\forall \gamma_1 > 0\ \exists \underline{\delta}_1 \text{ s.t. } \forall \delta_1 \geq \underline{\delta}_1, \quad \frac{\ln \delta_1}{\ln \delta_2(\delta_1)} \leq \gamma_1$$

For arbitrary ϵ, $\eta \in (0,1)$, set $\gamma_1 = \frac{\ln(1-\eta)}{\ln \epsilon}$, and (1) follows. □

Proof of Lemma 2.2:

1.

$$\begin{aligned}\delta_2(\delta_1)^{T(\delta_1)} &= \delta_2(\delta_1)^{\left[\frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}+[T(\delta_1)-\frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}]\right]} \\ &= \delta_2(\delta_1)^{[T(\delta_1)-\frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}]} . \frac{\mu_0}{\bar{\mu}}^{\frac{\ln \delta_2(\delta_1)}{\ln(1-\Delta(\delta_1))}}.\end{aligned}$$

Since $\lim_{\delta_1\to 1}(T(\delta_1) - \frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}) = 0$, therefore $\lim_{\delta_1\to 1} \delta_2(\delta_1)^{[T(\delta_1)-\frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}]} = 1$.
Next, the limit of the exponent of the second term as δ_1 goes to 1, which by L'Hospital rule, is given by:

$$\lim_{\delta_1\to 1} \frac{\ln \delta_2(\delta_1)}{\ln(1-\Delta(\delta_1))} = \frac{q}{d}$$

It follows that

$$\begin{aligned}\lim_{\delta_1\to 1} \delta_2(\delta_1)^{T(\delta_1)} &= \lim_{\delta_1\to 1} \delta_1^{[T(\delta_1)-\frac{\ln(\frac{\mu_0}{\bar{\mu}})}{\ln(1-\Delta(\delta_1))}]} \lim_{\delta_1\to 1} (\frac{\mu_0}{\bar{\mu}})^{\frac{\ln \delta_2(\delta_1)}{\ln(1-\Delta(\delta_1))}} \\ &= (\frac{\mu_0}{\bar{\mu}})^{\frac{q}{d}}\end{aligned}$$

□

2.

$$\begin{aligned}\lim_{\delta_1\to 1} \delta_1^{T(\delta_1)} &= \lim_{\delta_1\to 1} \delta_2(\delta_1)^{T(\delta_1)\frac{\ln \delta_1}{\ln \delta_2(\delta_1)}} \\ &= \lim_{\delta_1\to 1} \delta_2(\delta_1)^{T(\delta_1)(\lim_{\delta_1\to 1} \frac{\ln \delta_1}{\ln \delta_2(\delta_1)})} \\ &= \lim_{\delta_1\to 1} \delta_2(\delta_1)^{T(\delta_1)m} \\ &= \frac{\mu_0}{\bar{\mu}}^{\frac{mq}{d}}\end{aligned}$$

□

Proof of lemma 2.3:

Given μ_0 and $\frac{\epsilon}{2}$, by Lemma 2.2 $\exists$ $\underline{\delta_1}$ such that $\forall$ $\delta_1 \geq \underline{\delta_1}$, $\delta_1^{T(\delta_1)} \leq \left(\frac{\mu_0}{\bar{\mu}}\right)^{\frac{mq}{d}} + \frac{\epsilon}{2}$. Set $\bar{\mu_0}$ such that $\left(\frac{\mu_0}{\bar{\mu}}\right)^{\frac{mq}{d}} = \frac{\epsilon}{2}$. Since $\left(\frac{\mu_0}{\bar{\mu}}\right)^x$ is strictly increasing in $\frac{\mu_0}{\bar{\mu}}$ when x is positive, $\forall$ $\mu_0 \leq \bar{\mu_0}$ and $\delta_1 \geq \bar{\delta_1}$, $\delta_1^{T(\delta_1)} \leq \epsilon$. □

Proof of Lemma 4.1:

If there exists $\hat{a}_2 \in A_2$ such that $g_1(\hat{a}_1, \hat{a}_2) > g_1(a_1^c, \hat{a}_2)$, then the lemma will be true. Suppose, by way of contradiction, that $g_1(\hat{a}_1, \hat{a}_2) > g_1(a_1^c, \hat{a}_2) \geq \bar{v}_1$. As the game is not a strongly-conflicting-interest game, there exists $v_2' > \underline{v}_2$ such that $(\bar{v}_1, v_2') \in V^*$. It follows that for some $\lambda \in (0, 1)$,

$\lambda(\bar{v}_1, v_2') + (1-\lambda)(g_1(\hat{a}_1, \hat{a}_2), g_2(\hat{a}_1, \hat{a}_2)) \gg (\bar{v}_1, \underline{v}_2)$, contradicting the definition of $\bar{v}_1$. Suppose for all $a_2 \in A_2$, $g_1(\hat{a}_1, a_2) \leq g_1(a_1^c, a_2)$. For the game not to be a strongly-dominant-action game, there must exists a_2' such that $g_1(\hat{a}_1, a_2') = g_1(a_1^c, a_2')$ and $g_1(\hat{a}_1, a_2') < \bar{v}_1$. Hence, the lemma is true.□

Proof of Lemma 5.2:

Since σ_1 , and σ_2 are identical to σ_1^N and σ_2^N in the first N-1 periods; therefore

$$|v_2(\sigma_1, \sigma_2) - v_2(\sigma_1^N, \sigma_2^N)| \leq \delta_2^N d$$

The desired inequality is obtained by setting N to be bigger than $\frac{\ln(\epsilon/d)}{\ln \delta_2}$. □

Proof of Lemma 5.3:

First notice that

$$\begin{aligned} v_2(\rho_1^N, \sigma_2^N) &= E_{\sigma_2}[v_2(\rho_1^N, s_2^N)] \\ &= |E_{\sigma_2}[\{1 - P_{\rho_1, s_2}(h_N(\hat{s}_1, s_2))\} E_{\rho_1}(v_2(s_1^N, s_2^N | s_1 \neq \hat{s}_1)) + P_{\rho_1, s_2} v_2(\hat{s}_1^N, s_2^N)]| \end{aligned}$$

As a result,

$$\begin{aligned} & |v_2((\rho_1^N, \sigma_2^N) - v_2(\hat{s}_1^N, \sigma_2^N)| \\ = \;& |E_{\sigma_2}[(1 - P_{\rho_1, s_2}(h_N(\hat{s}_1, s_2))) E_{\rho_1}(v_2(s_1^N, s_2^N | s_1 \neq \hat{s}_1)) - v_2(\hat{s}_1^N, s_2^N)]| \\ \leq \;& \epsilon |E_{\sigma_2}[E_{\rho_1}(v_2(s_1^N, s_2^N | s_1 \neq \hat{s}_1)) - v_2(\hat{s}_1^N, s_2^N)]| \\ \leq \;& \epsilon(1 - \delta_2^N) d \end{aligned}$$

The desired inequality can be obtained by setting $\epsilon = \frac{\xi}{(1-\delta_2^N)d}$. □

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